Your Money or Your Life: A Prescriptive Model for Health, Safety, and Consumption Decisions

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In this paper we develop a conceptual framework and model for valuing risks to an individual's health and life and to support decision making about investments in health, quality of life, and safety. Our treatment of health risks in the model builds on the popular quality-adjusted-life-year (QALY) framework that balances health quality and length of life issues. We extend this framework to consider financial concerns as well as health quality and length of life. Our model considers uncertainty in income and health and incorporates the decision maker's ability to adjust consumption over time in response to changes in expectations about health and income. We use this model to study the optimal tradeoffs between financial gains or losses and improvements or reductions in health or longevity and apply it in two example medical decision problems.

Key words: decision analysis; QALYs; consumption health and safety decisions; valuing health outcomes; time preference; value tradeoffs
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1. Introduction

Many activities, including taking a cholesterol-lowering medication, having hip-replacement surgery, or purchasing a safer car, can be viewed as making an investment of time and/or money in future health, quality of life, or safety. These decisions are thorny because they involve difficult tradeoffs (e.g., how much should I pay to reduce the probability of dying in an accident?), because they involve many interconnected decisions and uncertainties, and because the consequences often span a long time horizon. In this paper, we develop a conceptual framework and model for valuing risks to an individual's health and life and to support decision making about investments in health, quality of life, and safety. Our orientation is prescriptive in that our goal is to develop a model that can be used to help people make these difficult decisions; we do not aim to describe how people typically make such choices.

Our treatment of health risks in the model builds on the popular quality-adjusted-life-year (QALY) framework that balances health quality and length of life issues. In the QALY model, an individual's lifespan is divided into T periods, here assumed to be equal in length, and the QALY score is given by

$$\text{QALYs} = \sum_{t=1}^{T} q_t, \tag{1}$$

where q_t is the "health related quality of life" index in period t. This index is scaled so that, in each period,

$q_t = 1$ corresponds to perfect health and $q_t = 0$ corresponds to a health state that is equivalent to death, meaning the individual would be indifferent toward spending the rest of his or her life in such a state or dying immediately. The QALY model is used to help individuals make medical decisions (e.g., should I have surgery?) and in cost-effectiveness analysis to study the efficacy of health interventions or policies. In individual applications, decisions are made to maximize expected QALYs. In cost effectiveness analysis, researchers typically measure cost effectiveness in terms of the cost per expected QALY gained where the QALYs are calculated for an "average" individual or reference case: interventions whose cost per QALY saved is less than a threshold value (e.g., $50,000 per QALY) are viewed as cost effective.

In this paper, we extend the QALY framework to consider tradeoffs between financial objectives and health states. The utilities in each period are the product of a QALY-like health-state index and a utility for consumption in that period and the overall utility function is of the form

$$\text{L-QALYs} = \sum_{t=0}^{T} q_t u_t(c_t), \tag{2}$$

where c_t denotes the consumption in period t and $u_t(c_t)$ denotes the utility associated with this level of consumption. The idea behind this multiplicative form for each period ($q_t u_t(c_t)$) is that good health allows one to enjoy consumption and vice versa.

We call these utilities *life-QALYs* or *L-QALYs* to distinguish them from *health QALYs*, which we will denote *H-QALYS*, given by Equation (1).

One of the challenges of working with a utility model of this form is accounting for the decision maker's (DM's) ability to adjust consumption over time in response to changes in health or income. In this paper, we focus on the case where the consumption utility functions ($u_t(c_t)$) have an exponential form. This simplifies the calculation of the optimal consumption levels and allows us to calculate derived utilities that represent the DM's preferences for income and health states, taking into account the ability to adjust consumption. We use this derived utility for health and income to study the optimal tradeoffs between financial gains or losses and improvements or reductions in health or longevity. In particular, we study the DM's "willingness to pay" (WTP) to reduce health or death risks as well as his or her "willingness to accept" (WTA) such risks. Our model can thus be used to calculate WTPs and WTAs for cost-benefit analyses that require tradeoffs between financial and health or safety considerations.[1]

Our interest in this problem is intrinsic: We want to improve our own thinking about investments in health and safety and were not satisfied with the available models. Howard's work (Howard 1980, 1984) is perhaps closest to ours in that he also has a prescriptive orientation and considers an individual making tradeoffs between financial and health objectives. The model in Howard (1980) assumes a utility function that depends on the DM's constant annual consumption and length of life; Howard (1984) added a QALY-like adjustment to the length of life. Howard's analyses assume that all financial receipts or payments are converted into constant consumption through the purchase of a (hypothetical) actuarially fairly priced annuity. In contrast, rather than assuming the existence of a fairly priced annuity, we assume that the DM borrows and lends to optimize consumption over time, adapting to changes in expectations about future income or health. The optimal consumption streams are generally not constant over time.

Shepard and Zeckhauser (1984) and Bleichrodt and Quiggin (1999) also studied utility models similar to (2). Shepard and Zeckhauser (1984) developed a continuous-time model in which income and death risk vary over time and the DM can save (but not borrow) to optimize consumption over time; they also considered a case with actuarially fair annuities.

The goal of their paper was to provide an indirect method for estimating WTPs for reductions in death risks at various ages and their model is deterministic except for the possibility of death at various ages. To simplify their analysis, Shepard and Zeckhauser assumed that the utilities for consumption are identical in each period and are discounted at the market interest rate. Bleichrodt and Quiggin (1999) studied a discrete-time "lifecycle model" like (2) that considered consumption and health states over time and also required consumption utilities to be constant over time. Their primary goal was to identify assumptions that make cost-effectiveness analysis (e.g., QALY modeling) consistent with cost-benefit analysis based on this lifecycle model. The conditions they identify—both consumption utilities and consumption must be constant over time—are, in their words, "quite stringent" (Bleichrodt and Quiggin 2002, p. 167).

In the next section, we describe our model and an illustrative example that we will use throughout the paper. In §3, we describe the solution of this consumption problem. In §4, we discuss the valuation of health and death risks and determine WTPs and WTAs for such risks. In §5, we illustrate the application of the model to two personal medical decisions. In §6, we conclude and discuss directions for additional research.

2. The Model and an Illustrative Example

In this section, we first describe the general form of the model and the exponential utility assumption that simplifies our analysis. We then present a numerical example that we will use to illustrate the model throughout the rest of the paper.

2.1. General Model

We assume that the DM's possible lifespan is divided into T periods, which for ease of exposition, are taken to be equal in length. The DM's lifetime utility is represented by

$$U(\mathbf{c}; \mathbf{q}) \equiv \sum_{t=0}^{T} q_t u_t(c_t), \tag{3}$$

where c_t is the DM's monetary consumption in period t and q_t is a health-state index that captures the DM's mental and physical ability to enjoy consumption; $\mathbf{c} = (c_0, \dots, c_T)$ and $\mathbf{q} = (q_0, \dots, q_T)$ are consumption and health-index streams.[2] The consumption utility functions u_t are assumed to be

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[1] See, e.g., Sox et al. (1988) for examples of the use of QALYs for individual decisions and Gold et al. (1996) for discussion of QALY-based cost effectiveness analysis. See Hammitt (2002), Dranove (2003), or Krupnick (2004) for recent comparisons of WTP-based cost-benefit analysis and QALY-based cost-effectiveness analysis.

[2] Although we will not pursue this in detail in this paper, this form of utility function can be justified by appealing to more primitive assumptions. First, we assume that the DM's preferences are additive independent over time, allowing us to write $U(\mathbf{c}; \mathbf{q})$ as the sum of utilities for individual periods. Second, we assume that for each period the DM's preferences for health states (q_t) are utility

strictly increasing, concave, and continuously differentiable: higher levels of consumption lead to greater utility, but at a decreasing rate. Like the "health-related quality of life" index in the H-QALY model (1), the health-state index q_t for period t is scaled so that death corresponds to $q_t = 0$ and perfect health corresponds to $q_t = 1$. The health and income streams continue through period T with death at time t_d represented by having $q_t = 0$ for all $t \geq t_d$. We will assume that $q_t \geq 0$ because negative q_ts would lead one to prefer less consumption to more. Note that we do not require the utility functions q_t to be identical in each period. One could incorporate discounting of future utilities by including the discount factor in u_t. Our L-QALY model (3) reduces to the H-QALY model (1) in the special case where the consumption utilities $u_t(c_t)$ are equal to 1.

Rather than assuming that the DM converts income into constant consumption through the purchase of a hypothetical fairly priced annuity (as Howard 1980, 1984 and others do), we will assume that the DM optimizes consumption by borrowing or lending over time with interest earned or paid at a constant rate r. While we will not place any constraints on the DM's borrowing over time, we require the DM to have a zero balance at the end of the planning horizon (time T) or, equivalently, we require the net present value (NPV) of the consumption stream to match the NPV of the income stream over the time horizon. The DM may die with unspent savings or owing money. As we will see with our example, the aversion to having low or negative consumption levels provides a strong incentive for savings and, if one earns money early in life and retires later (as in our example), the DM typically dies with unspent savings. Thus, if we were to add a borrowing constraint, it would typically not be binding in such cases.

The DM's income stream $\tilde{\mathbf{x}} = (\tilde{x}_0, \ldots, \tilde{x}_T)$ and health-index stream $\tilde{\mathbf{q}} = (\tilde{q}_0, \ldots, \tilde{q}_T)$ are both uncertain. In period t, the DM learns his income x_t and health index q_t and then chooses his consumption level c_t. Given an uncertain income stream $\tilde{\mathbf{x}}$ and health-index stream $\tilde{\mathbf{q}}$, the DM will choose a consumption stream $\tilde{\mathbf{c}} = (\tilde{c}_0, \ldots, \tilde{c}_T)$ to solve the following maximization problem:

$$\mathcal{U}(\tilde{\mathbf{x}}, \tilde{\mathbf{q}}) \equiv \max_{(\tilde{c}_0, \ldots, \tilde{c}_T)} \mathrm{E}\left[\sum_{t=0}^{T} \tilde{q}_t u_t(\tilde{c}_t)\right] \qquad (4)$$

independent of consumption (c_t) and that the DM is indifferent about consumption (c_t) when in a utility state equivalent to death ($q_t = 0$). Following an argument similar to that of Miyamoto et al. (1998), this second assumption allows us to write the period utilities as a product $q_t u_t(c_t)$. Additive independence and utility independence are defined and discussed in detail in Keeney and Raiffa (1976); applications to QALY models are discussed in Bleichrodt and Quiggin (1999) and Hazen (2004).

subject to

$$\sum_{t=0}^{T} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{x_t}{(1+r)^t},$$

where the constraint captures the requirement (discussed in the previous paragraph) that the NPV of the DM's consumption stream must match the NPV of his income stream over the planning horizon; $\tilde{c}_t$ and $\tilde{x}_t$ are random variables and the constraint requires these NPVs to match in every state of the world. The derived utility function $\mathcal{U}(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ describes the expected utility resulting from an uncertain income stream $\tilde{\mathbf{x}}$ and health-index stream $\tilde{\mathbf{q}}$ and can be used to evaluate alternative health and income profiles. The DM should choose between alternative health-income profiles $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ to maximize derived utility $\mathcal{U}(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$.

Following the derivation in the appendix, we find that the first-order conditions for the DM's consumption optimization problem (4) require that the optimal consumption levels c_t^* satisfy

$$(1+r)^t q_t u_t'(c_t^*) = (1+r)^\tau \mathrm{E}_t[\tilde{q}_\tau u_\tau'(\tilde{c}_\tau^*)] \qquad (5)$$

for all $\tau > t$ and for every possible period-t state. Here $\mathrm{E}_t[\cdot]$ denotes expectations conditional on the time t state of information, u_t' denotes the derivative of the period-t consumption utility function u_t, and $\tilde{c}_\tau^*$ denotes the currently uncertain optimal consumption in period τ. The optimal period-t consumption level c_t^* is chosen (knowing q_t and x_t) so the marginal utility in period t (on the left of Equation (5)) matches the expected future marginal utility (on the right of Equation (5)) in all periods with each period's marginal utilities adjusted for the time value of money. Because u_t was assumed to be continuously differentiable and concave, the objective function in (4) is continuous and concave in $(\tilde{c}_0, \ldots, \tilde{c}_T)$. Thus, if we find a consumption stream satisfying the first-order conditions (5) and the feasibility constraint in (4), we have found an optimal solution.

2.2. The Additive-Exponential Utility Model

For general utility functions, this consumption optimization problem (4) can be quite difficult to solve as it requires solving a nonlinear programming problem with decision variables corresponding to the amount consumed in each period, for each state of the world. However, the calculations simplify greatly if we assume that the DM has an exponential form for the consumption utility,

$$u_t(c_t) = k_t(\alpha_t - \exp(-c_t/\rho_t)). \qquad (6)$$

Here the period-t utility weight k_t and period-t consumption risk tolerance ρ_t are both assumed to be positive and represent the DM's time and risk prefer-

Figure 1   An Exponential Consumption Utility
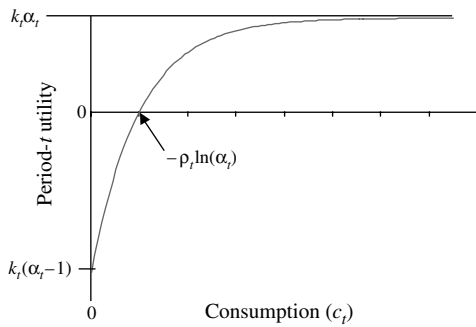
ences, respectively. The α_t parameter is used to control the "zero-utility level" of consumption, meaning the level of consumption that leads to a period-t utility of 0, regardless of the health state. For example, setting $\alpha_t = 1$ will imply that $u_t(0) = 0$. In general, the zero level of consumption is $-\rho_t \ln(\alpha_t)$ with the zero level varying from 0 to ∞ as α_t goes from 1 toward 0. A plot of a utility function represented by (6) is shown in Figure 1.

With this form for $u_t(c_t)$, the overall utility function (3) may be rewritten as

$$U(\mathbf{c}; \mathbf{q}) = \sum_{t=0}^{T} k_t \alpha_t q_t - \sum_{t=0}^{T} k_t q_t \exp(-c_t/\rho_t), \qquad (7)$$

which we refer to as an *additive-exponential utility function*. The first term in (7) is independent of consumption and can be interpreted as the utility that would be obtained if one had no financial constraints. If we take $k_t = 1/\alpha_t$, then the first term in (7) will be the H-QALY value. The second, subtracted term in (7) can be interpreted as a "finite wealth penalty" representing the utility loss associated with having finite wealth and, hence, limited consumption: it is nonnegative and approaches zero for large consumption levels.

## 2.3.   An Illustrative Example

We will illustrate the results of the paper by considering a hypothetical example of a 30-year-old professor named Jack. To keep the example simple, we will assume that Jack is in perfect health with a health-state index $q_t = 1.0$ until he dies and that his annual death probabilities follow the standard mortality probabilities for white males (Arias 2004). With these assumptions, Jack's expected remaining life and his expected H-QALY score are both 44.15 years. We will consider consumption decisions on an annual basis and take the planning horizon T to be 70 years, corresponding to when Jack is 100 years old.

Jack begins his career at age 30 by taking a tenure-track position at a prestigious university that pays $60,000 (inflation adjusted) per year. In 10 years

(at age 40), he goes up for tenure. If he gets tenure, his pay increases by $10,000 per year. If he does not get tenure, he must find a new job elsewhere and his pay drops by $10,000 per year. His probability of getting tenure is 0.50. Ten years after his initial tenure decision, he has a 50% chance of being promoted for which he will receive a raise of $10,000 per year at the prestigious university or $5,000 per year at the other university. In all scenarios, Jack retires at age 65 and earns no additional income. These possible income streams are shown in Figure 2. Jack's savings are assumed to earn interest at a real rate $r = 2\%$ per year. We also assume that Jack does not update his probabilities of getting tenure or promoted until the uncertainty is resolved and his probabilities for death do not depend on whether Jack receives tenure or is promoted.

We will assume that Jack has an additive-exponential utility function. The consumption risk tolerances ρ_t are assumed to be $10,000 per year, which implies that Jack is approximately indifferent to accepting or refusing a gamble that would increase his consumption by $10,000 ($=\rho_t$) per year (for example, from $40,000 per year to $50,000) or decrease it by $5,000 ($=\rho_t/2$) per year (e.g., from $40,000 to $35,000), with equal probabilities. We set the constant terms α_t to be 0.3679 in each year which implies that Jack has a zero-utility level of $10,000. In this example, we will assume utility weights $k_t = 1/\alpha_t$ in each period, which means that there is no discounting of future utilities and the expectation of the first term in (7) is Jack's expected H-QALY score, 44.15. With the optimal consumption stream (to be determined later), the expectation of the second term in (7) is 1.97 which implies that Jack's limited wealth costs him approximately two H-QALYs in terms of his overall utility.

Figure 2   Income Scenarios for the Example
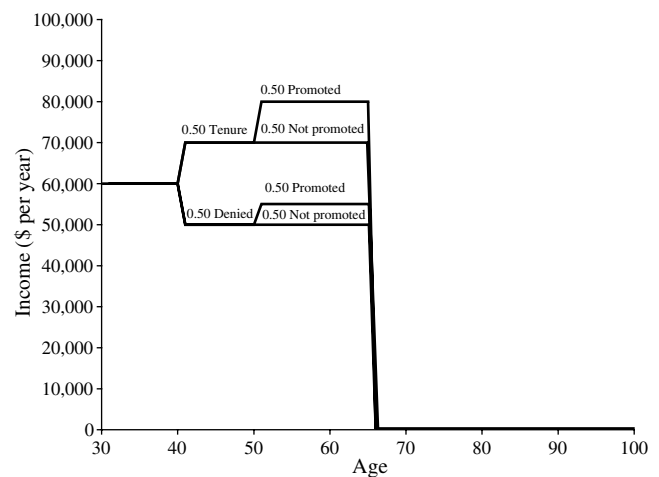
3. Solving the Optimal Consumption Problem

We now consider the problem of determining the optimal consumption stream given an additive-exponential utility function. To see how this utility function simplifies the analysis, consider the first-order conditions (5) for the DM's optimization problem with this utility function. If we change income or health risks, we can write the new optimal consumption stream as the sum of the old optimal consumption stream and an adjustment as $\tilde{\mathbf{c}}^{\text{new}} = \tilde{\mathbf{c}}^{\text{old}} + \tilde{\mathbf{c}}^{\text{adj}}$ and the first-order conditions (5) for determining the optimal consumption stream become

$$(1+r)^t q_t u'_t(c_t^{\text{old}}) \exp(-c_t^{\text{adj}}/\rho_t)$$
$$= (1+r)^\tau E_t[\tilde{q}_\tau u'_\tau(\tilde{c}_\tau^{\text{old}}) \exp(-\tilde{c}_\tau^{\text{adj}}/\rho_t)] \quad \text{for all } \tau > t.$$
(8)

Thus, with this form, the consumption adjustment enters as a multiplier in the first-order conditions. With many changes in health index or income streams, we can determine the optimal consumption adjustment c_t^{adj} that restores the balance in (8) quite easily without reconsidering the optimization problem (4). Moreover, with independent changes to the income and health streams, the expectation in (8) factors, and the consumption adjustments will be additive and can be determined separately.

We can use these properties of the additive-exponential utility function to decompose the optimal consumption problem (4) into three simpler subproblems that can be solved separately. Specifically, we

(1) Determine the optimal consumption stream given no income and perfect health over the length of the planning horizon, $\tilde{x}_t = 0$ and $\tilde{q}_t = 1$ for all t, meaning that the DM lives in perfect health to T (age 100 in the example) with no income. While this is not a realistic scenario in itself, this base consumption stream can be seen as representing the DM's time preferences in the solution of the optimal consumption problem. We discuss this subproblem in §3.1.

(2) Determine the adjustment to consumption associated with the DM's uncertain income stream $\tilde{\mathbf{x}}$. As we will see in §3.2, this income adjustment is straightforward to calculate and is independent of the health-index stream.

(3) Determine the adjustment to consumption associated with the DM's uncertain health-index stream $\tilde{\mathbf{q}}$. As we will see in section §3.3, this adjustment is also straightforward to calculate and is independent of the income stream.

We can then add these two adjustments to the base consumption level to determine the optimal consumption stream with both income and health risks. The three components of the consumption stream

Figure 3   Components of the Optimal Consumption Stream in the Example
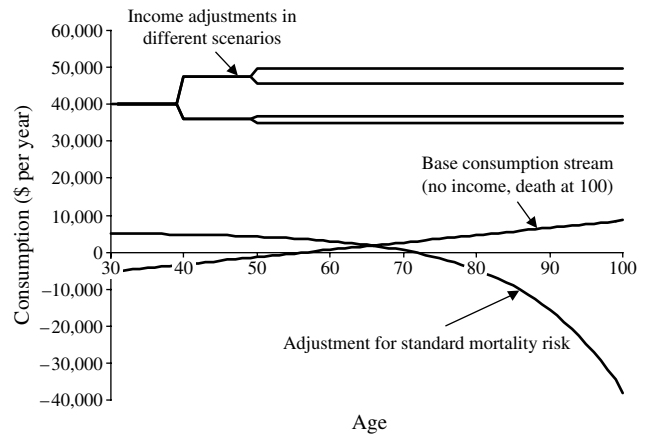
in the example are shown in Figure 3 and will be discussed in the next three subsections. The total consumption is shown in Figure 4. The income adjustment and total consumption both depend on the income scenario (i.e., whether Jack gets tenure and/or gets promoted) and the scenarios here reflect the scenarios indicated in Figure 1.

In addition to simplifying the solution of the consumption problem, the additive-exponential utility function leads to analytic forms for the derived utility function $\mathcal{U}(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ of Equation (4). This derived utility function can be used to determine the DM's WTP and WTA values for changes in health or death risks that reflect the optimal changes in consumption. This decomposition also helps clarify how the different factors—interest rates, income, health, and mortality risks—should affect the individual's consumption decisions.

3.1. Solving the Base Consumption Problem

In this section, we consider the problem of determining the optimal consumption stream given no income

Figure 4   Total Consumption in the Different Income Scenarios in the Example
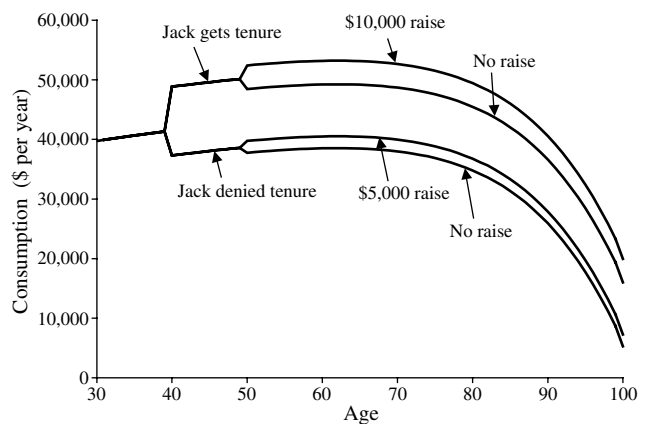
and perfect health, $\tilde{x}_t = 0$ and $\tilde{q}_t = 1$ for all t, given a utility function with the additive exponential form. As indicated earlier, this problem is not of direct interest in itself but is part of the decomposed procedure for determining the optimal consumption stream. In this case, the first-order conditions (5) imply that the base consumption stream c_t^0 satisfies

$$(1+r)^t k_t / \rho_t \exp(-c_t^0/\rho_t)$$
$$= (1+r)^\tau k_\tau / \rho_\tau \exp(-c_\tau^0/\rho_\tau) \quad \text{for all } \tau \text{ and } t. \quad (9)$$

Combining this with the budget constraint in (4) with zero income, we find that the base consumption levels are

$$c_t^0 = \rho_t \ln\left(\frac{k_t(1+r)^t}{\rho_t}\right)$$
$$- \frac{\rho_t}{R_0}\left(\sum_{\tau=0}^{T}\frac{1}{(1+r)^\tau}\ln\left(\frac{k_\tau(1+r)^\tau}{\rho_\tau}\right)\right), \quad (10)$$

where

$$R_0 \equiv \sum_{t=0}^{T}\frac{\rho_t}{(1+r)^t}. \quad (11)$$

Examining (10), we find that, as one would expect, the period-t base consumption level (c_t^0) is increasing in the period-t utility weight (k_t) and is decreasing in other period's utility weights (k_τ, for $\tau \neq t$). Note that the optimal consumption levels are independent of the α_t parameters in the DM's utility function (7); this follows from the fact that these parameters do not interact with the consumption terms in the utility function.

Applying Equation (10) in our example, we find the base consumption stream shown in Figure 3. In the figure, we see that this base consumption stream is increasing over the planning horizon. Assuming zero income and no health/death risks, it is optimal to consume negative amounts in the early years (which means that the DM saves money and earns interest) and consume larger positive amounts later. This base consumption stream also reflects the assumption in the example that utilities are not discounted. Examining Equation (10), we see that if we had discounted utilities over time at the interest rate by taking $k_t \propto (1+r)^{-t}$ while assuming a constant consumption risk tolerance (ρ_t) over time, we would have constant base consumption (c_t^0). The base consumption stream thus depends on the time preference parameters k_t in the utility function. We will see below that this is the only component of the optimal consumption stream affected by these parameters.

3.2. Evaluating Income Streams
Next, we consider the effects of changing income on the DM's consumption and utility. Before considering the case of an uncertain income stream, let us first consider the effects of a deterministic change in income. Suppose that the DM initially has income stream $\tilde{x}^{\text{old}}$ and health-index stream $\tilde{q}$ and, given this, follows optimal consumption strategy $\tilde{c}^{\text{old}}$. Now suppose that we add a deterministic stream $x^{\text{adj}} = (x_0^{\text{adj}}, \ldots, x_T^{\text{adj}})$ to the DM's income and assume a new consumption stream $\tilde{c}^{\text{new}} = \tilde{c}^{\text{old}} + c^{\text{adj}}$, where

$$c_t^{\text{adj}} = \frac{\rho_t}{R_0}\left(\sum_{\tau=0}^{T}\frac{x_\tau^{\text{adj}}}{(1+r)^\tau}\right) \quad (12)$$

and R_0 is as defined in Equation (11). One can verify that this consumption adjustment is feasible (the net present value of c^{adj} is equal to the net present value of x^{adj}) and $\tilde{c}^{\text{new}}$ satisfies the first-order conditions given by Equation (8). Thus, the optimal response to a deterministic change in the income stream is to share the net present value (NPV) of the additional income stream over time in proportion to the period consumption risk tolerances. Note that the optimal consumption adjustments $\tilde{c}_t^{\text{adj}}$ are independent of the DM's initial income stream $\tilde{x}^{\text{old}}$, health-index stream $\tilde{q}$, and the k_t and α_t parameters of the utility function.

Now suppose that the DM faces an income gamble $\tilde{x}^{\text{adj}}$ that is resolved immediately (before any consumption decisions are made) and is independent of the DM's initial income stream $\tilde{x}^{\text{old}}$ and health-index stream $\tilde{q}$. Once this gamble is resolved, the additional income $\tilde{x}^{\text{adj}}$ is deterministic and consumption should be adjusted as given by Equation (12). The derived utility $\mathcal{U}(\tilde{x} + x^{\text{adj}}, \tilde{q})$ (defined in Equation (4)) associated with such a change in income can be determined by substituting the adjusted consumption (12) into the utility formula (7) and taking expectations. The result is

$$\mathcal{U}(\tilde{x} + x^{\text{adj}}, \tilde{q}) = a^{\text{old}} - b^{\text{old}}\exp\left(-\frac{1}{R_0}\sum_{t=0}^{T}\frac{x_t^{\text{adj}}}{(1+r)^t}\right), \quad (13)$$

where

$$a^{\text{old}} \equiv \left(\sum_{\tau=0}^{T}k_\tau \alpha_\tau E_0[\tilde{q}_\tau]\right), \quad \text{and} \quad (14)$$

$$b^{\text{old}} \equiv \left(\sum_{\tau=0}^{T}k_\tau E_0[\tilde{q}_\tau \exp(-\tilde{c}_\tau^{\text{old}}/\rho_\tau)]\right). \quad (15)$$

Thus, the derived utility for changes in income has the form of an exponential utility function defined on the NPV of the income change with an *effective risk tolerance* R_0 equal to the NPV of the period consumption risk tolerances. The scaling constants a^{old} and b^{old} are the expectations of the first and second terms in Equation (7) with the initial consumption stream: a^{old} represents the utility of life without wealth constraints (i.e., the H-QALY score, if the utility parameters are scaled appropriately) and b^{old} represents the

"finite wealth penalty" at the base income level. The exponential term in the derived utility (13) reduces this penalty as the NPV of the income adjustment increases. While a^{old} and b^{old} depend on the DM's initial income stream $\tilde{x}^{old}$ and health-index stream $\tilde{q}$, changes in a^{old} and b^{old} lead to an affine change to the derived utility for income. Thus, the old income stream $\tilde{x}$ and health index stream $\tilde{q}$ do not affect the DM's preferences for incremental, immediately resolved gambles.

Let us apply these ideas in our example. With up to 70 years of life remaining, the effective risk tolerance R_0 for Jack's derived utility function (13) is equal to the present value of a 70-year stream of $\rho_t = \$10,000$ in each year. At the assumed discount rate of 2%, this is $384,986. Thus, Jack should be roughly indifferent toward accepting a 50-50 gamble in which he can win $384,986 ($=R_0$) or lose $192,493 ($=R_0/2$). Following Equation (12), any amount he wins or loses would be spread over the remainder of his life, increasing or decreasing his annual consumption by 2.60% ($=\rho_t/R_0$) of the NPV of the change in income.

With this additive-exponential utility function, we can also easily evaluate risky income streams whose uncertainties are not resolved immediately using the valuation procedure developed in Smith (1998). While that paper did not consider uncertainty about health or death risks, if the additional income stream being evaluated $\tilde{x}^{adj}$ is independent of the health-index stream $\tilde{q}$ and any prior income stream $\tilde{x}^{old}$, the same procedure applies, as discussed in the appendix. The valuation procedure is a slight modification of the standard decision tree "roll back" procedure and proceeds as follows. First, place the nodes in order corresponding to the time at which the uncertainties are resolved, with later decisions and uncertainties appearing further to the right in the tree. Next, calculate NPVs for each endpoint in the tree by discounting all cash flows using the interest rate r; label these final endpoint values v_T. We then "roll back" the tree, working from right to left. Decision nodes are handled in the usual way: Given a choice among different income streams at some point in the tree, choose the alternative that leads to the maximum successor rollback value. Given a chance node representing an uncertainty resolved in period t with successor values $\tilde{v}_t$, the rollback value, v_{t-1}, is the certainty equivalent given by using an exponential utility function with *effective risk tolerance* R_t:

$$v_{t-1} = -R_t \ln(E_t[\exp(-\tilde{v}_t/R_t)]), \qquad (16)$$

where

$$R_t \equiv \sum_{\tau=t}^{T} \frac{\rho_\tau}{(1+r)^\tau}. \qquad (17)$$

The value v_0 given at the root of the tree is the DM's *present certainty equivalent value* (PCEV): the DM should be indifferent between receiving the risky income stream $\tilde{x}^{adj}$ or an immediate lump-sum value v_0 because they give the same expected utility. The effective risk tolerance R_t used to evaluate uncertainties resolved in period t is the NPV of subsequent consumption risk tolerances and reflects the DM's ability to share the anticipated increases or decreases in income over subsequent time periods; the specifics of this sharing rule will be discussed shortly.

We can illustrate this procedure by evaluating Jack's income prospects shown in Figure 2. The calculations for the valuation procedure are shown in Figure 5. First, we calculate NPVs for each possible income scenario; these are shown at the endpoints of the tree. Rolling the tree back from right to left, the first uncertainty encountered is the node corresponding to whether Jack will be promoted in 20 years. This is evaluated using a risk tolerance $R_{20} = \$218,202$ and yields certainty equivalents of $1,759,058 or $1,420,306, respectively, in the cases where Jack is granted tenure or not. We then evaluate the tenure uncertainty using $R_{10} = \$293,364$ and find a PCEV of $1,543,283: Jack should be indifferent to receiving his uncertain future income stream and receiving $1,543,283 as a lump sum, immediately with certainty.

This PCEV reflects both risk and delay premiums. If Jack's income gamble had been resolved immediately, we would calculate the effective certainty equivalents using the current effective risk tolerance $R_0 = \$384,946$ and find a certainty equivalent of $1,554,430, instead of the $1,543,283 calculated using the time-varying effective risk tolerances. The difference between these two amounts is $11,147 and can be interpreted as a delay premium, the loss due to the delayed resolution of uncertainty. If the gamble were resolved immediately, Jack would adjust

Figure 5 **Valuing the Risky Income Streams in the Example**
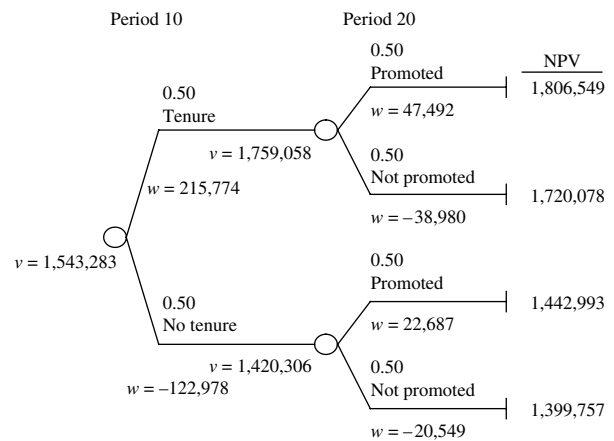
his consumption immediately depending on how the gamble turned out. With the delayed gamble, he does not adjust consumption until after the uncertainties are resolved and winds up with an allocation of consumption that is ex post suboptimal. The difference between the expected NPV of his future income ($1,592,344) and the certainty equivalent calculated using R_0 everywhere ($1,554,430) is $37,914; this can be interpreted as a risk premium in the usual way as a loss due to uncertainty in future income. These risk and delay premiums are discussed in more detail in Smith (1998).

How does one adjust consumption in light of these changes in income? To study this, let us define the *period-t windfall* as $\widetilde{w}_t = \widetilde{v}_t - \widetilde{v}_{t-1}$ for $t > 0$ and $w_0 = v_0$. Intuitively, these windfalls describe the change in the PCEV of the income stream over time. These changes are measured in present value terms and, by construction, the sum of the windfalls along a path through a decision tree is equal to the NPV associated with the endpoint of the tree $\widetilde{v}_T$. With the additive-exponential utility, Smith (1998) shows that the optimal consumption adjustments are analogous to the adjustment specified in Equation (12) and are

$$\tilde{c}_t^{\mathrm{adj}} = \rho_t \sum_{\tau=0}^{t} \frac{\widetilde{w}_\tau}{R_\tau}. \tag{18}$$

The windfalls for the example are noted in Figure 5. If Jack receives tenure, his certainty equivalent increases from $1,543,283 to $1,759,058 for a windfall of $215,774. In this scenario, he should increase his consumption by $(\rho_t/R_{10})\$215,774 = (\$10,000/\$293,364) \times \$215,774 = \$7,355$ in each subsequent year. Being denied tenure leads to a decrease in consumption of $4,192 in each year. These annual consumption adjustments are shown in Figure 3, with the tree structure in the consumption adjustments reflecting the same scenarios as in Figure 2.

3.3. Consumption Adjustments for Health and Death Risks

How do health and death risks affect the DM's optimal consumption and utility? First, suppose that we reduce the expected health-state index in period t by a factor of γ_t, so that the new period-t expected health-state index becomes

$$\mathrm{E}_0[\tilde{q}_t^{\mathrm{new}}] = (1 - \gamma_t)\mathrm{E}_0[\tilde{q}_t^{\mathrm{old}}] \tag{19}$$

and all other period's health-state indices are unchanged. In this case, it is straightforward to determine the consumption adjustment due to this change in health expectations. If we assume a new consumption stream $\tilde{\mathbf{c}}^{\mathrm{new}} = \tilde{\mathbf{c}}^{\mathrm{old}} + \mathbf{c}^{\mathrm{adj}}$ where

$$c_\tau^{\mathrm{adj}} = \begin{cases} -\rho_\tau \ln((1 - \gamma_t)^{[\rho_t/(1+r)^t]/R_0 - 1}), & \tau = t, \\ -\rho_\tau \ln((1 - \gamma_t)^{[\rho_t/(1+r)^t]/R_0}), & \tau \neq t, \end{cases} \tag{20}$$

after some algebra, we can verify that this new $\tilde{c}^{\mathrm{new}}$ is feasible in that the adjustments $\mathbf{c}^{\mathrm{adj}}$ have NPV zero and are optimal because they satisfy the first-order conditions (8) with the revised expected health-state indices (19). Examining (20), we see that a decrease in the expected health index in period t leads to a decrease in consumption in period t and an increase in other periods. This consumption adjustment depends only on the risk tolerance parameters of the DM's utility function. Moreover, the magnitude of the period-t adjustment does not depend on the base health index or income stream and adjustments for changes in health indices in different periods can be added together.

We can illustrate these consumption adjustments by considering the standard mortality risks in our example. From the mortality table, we find that there is a 0.0788 chance that Jack will be dead before he turns 50 and therefore his expected health index for that period is 0.9212. Reducing the expected health index from 1.0 (as previously assumed) to 0.9212 using Equation (20), we find that Jack should adjust consumption down in that year by $-\rho_\tau \ln(1 - 0.0788)^{((\rho_{20}/R_0)-1)} = -\$10\mathrm{k} \times \ln(1 - 0.0788)^{(($10\mathrm{k}/$385\mathrm{k})-1)} = \$806.85$ and adjust consumption up in other years by $-\rho_\tau \ln(1 - 0.07884)^{(\rho_{20}/R_0)} = -\$10\mathrm{k} \times \ln(1 - 0.0788)^{($10\mathrm{k}/$385\mathrm{k})} = \$14.35$. By accumulating similar consumption adjustments for each year, we arrive at the total consumption adjustment for the standard mortality risks shown in Figure 3. Here we see that these mortality risks lead Jack to increase consumption early in life and decrease consumption later.

Adding the consumption adjustments for mortality and income to the base consumption level, we arrive at the optimal consumption streams shown in Figure 4. These optimal consumption streams reflect the increasing trend of the base consumption level (reflecting Jack's time preferences and interest earned on savings), an upward shift provided by Jack's uncertain income stream, and accelerating downward adjustments due to Jack's mortality. As indicated earlier, with this optimal consumption stream, Jack's overall expected utility or expected L-QALY score is 42.18, reflecting an expected H-QALY score of 44.15 less a "finite wealth penalty" of 1.97.

4. Valuing Health and Death Risks

Having discussed the effects of income and health risks on consumption, we now turn to the problem of valuing health and death risks. We begin by studying the marginal value of an L-QALY (in §4.1) and then turn to the valuation of changes in death risks (in §4.2) and health states (in §4.3).

4.1. The Marginal Value of an L-QALY

Because utilities are measured in L-QALYs, the derived utility function for income (13) captures the

tradeoff between health and income by describing the value of additional wealth in terms of L-QALYs. Given a base income stream of $\tilde{\mathbf{x}}$ and holding the heath-state stream $\tilde{\mathbf{q}}$ constant, the marginal effect of a change w_0 in period-0 income (effectively a change in wealth) is given by the derivative of derived utility function in (13):

$$\frac{\partial \mathscr{U}(\tilde{\mathbf{x}} + w_0, \tilde{\mathbf{q}})}{\partial w_0} = \frac{b^{\text{old}}}{R_0} \exp(-w_0/R_0), \qquad (21)$$

where b^{old} and R_0 are defined in Equations (15) and (11). The reciprocal of this,

$$Q_0 = \frac{R_0}{b^{\text{old}}} \exp(w_0/R_0), \qquad (22)$$

can be interpreted as the marginal value of an L-QALY, describing the DM's willingness to trade money for L-QALYs.

Figures 6 and 7 show the derived utility and marginal value of an L-QALY as wealth changes in our example. The bold curves in each figure show the utilities and marginal value given the base assumptions for the example; the lighter curves consider alternative consumption risk tolerances. In Figure 6, we see the exponential form for the derived utility, which asymptotically approaches the expected H-QALY score as wealth increases and the finite wealth penalty decreases. In Figure 7, we see that the exponential form of (22) leads the marginal value of an L-QALY to increase rapidly with increases in wealth. Intuitively, an increase in income moves Jack up his utility curve (shown in Figure 6) and the utility curve becomes flatter and the marginal utility for income decreases. Thus, changes in wealth have less impact on his L-QALY score and Jack will demand more money to sacrifice L-QALYs and will be willing to pay more to gain additional L-QALYs.

Comparing the marginal curves for different consumption risk tolerances, we see that the marginal values are quite sensitive to changes in these preference parameters. The bold curves show utilities
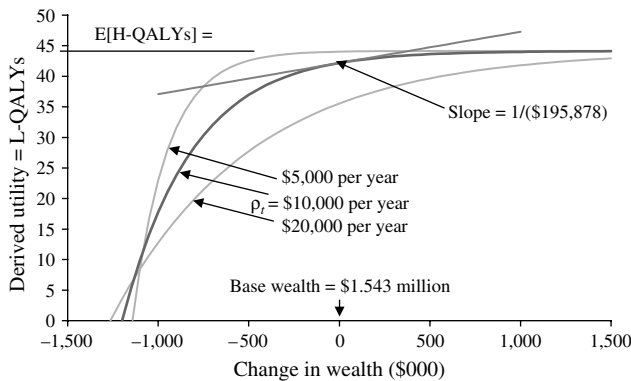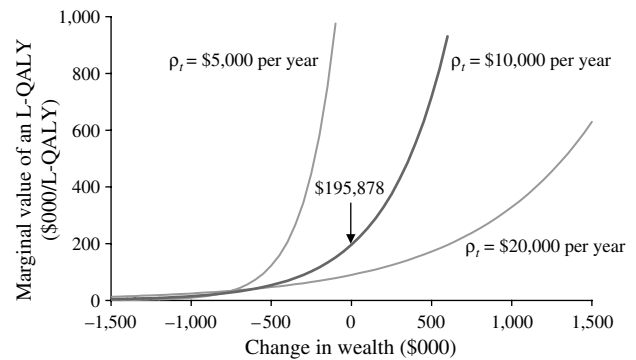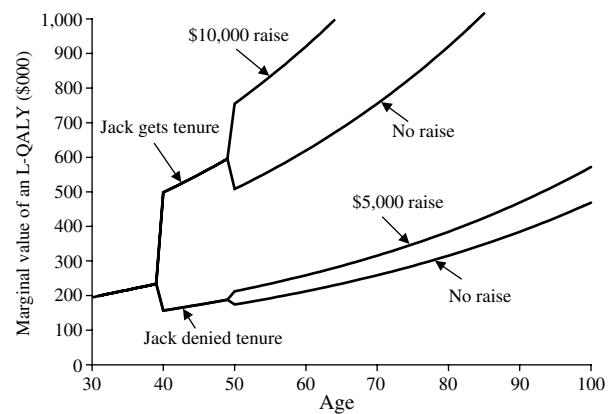
Figure 7   The Marginal Value of an L-QALY as a Function of Wealth

and marginal values assuming the base consumption parameters, including a consumption risk tolerance of $10,000 per year and a zero-utility level of consumption of $10,000 per year. The light curves show the utilities and marginal values with consumption risk tolerances of $5,000 and $20,000 per year, holding the zero-level of consumption constant. With a greater risk tolerance, the utility curves in Figure 6 become less flat and the marginal value of an L-QALY decreases substantially. At the base wealth level, the marginal values of an L-QALY are $1,641,291, $195,878, and $89,718, respectively, for consumption risk tolerances of $5,000, $10,000, and $20,000 per year. While this is a broad range of risk tolerances, this extreme sensitivity points to a need to assess these preference parameters carefully.

Figure 8 shows how the marginal value of an L-QALY changes over time in the example, with the different paths corresponding to the different income scenarios. Here we see that increases or decreases in income expectations lead to significant changes in the marginal value of an L-QALY: Jack's marginal value of an L-QALY at age 40 if he receives tenure ($498,216) is more than three times the marginal value at the same age if he does not receive tenure ($157,011). This dramatic change reflects the exponential wealth

Figure 6   Derived Utility as a Function of Wealth

Figure 8   The Marginal Value of an L-QALY Over Time

sensitivity shown in Equation (22) and illustrated in Figure 7. Between these changes in marginal values due to changes in expected income, the marginal value of an L-QALY increases at exactly the interest rate r. As shown in the appendix, this follows from the first-order condition for the consumption optimization problem (4) and holds for nonexponential consumption utilities, as well as exponential utilities. The intuition is that the DM will choose a consumption quantity at time t that equates the marginal gain in utility associated with consuming more in that period with the marginal expected loss in utility from reduced savings and consequent reduction in consumption in the future. In an optimal solution, this leads to marginal utilities for wealth to be declining over time at interest rate r or, equivalently, the expected marginal value of an L-QALY to be increasing at rate r.

4.2. Valuing Death Risks

We now turn to the problem of determining how much one should be willing to pay to avoid or be paid to accept changes in death risks. Let p_t denote the probability of dying in period t conditional upon living through period $t-1$. The probability of being alive in period t is given in terms of these conditional probabilities as

$$\prod_{\tau=0}^{t}(1-p_\tau). \tag{23}$$

Now suppose that we have a DM with income stream $\tilde{\mathbf{x}}$, initial health-index stream $\tilde{\mathbf{q}}^{\text{old}}$, and optimal consumption stream $\tilde{\mathbf{c}}^{\text{old}}$. If we change the probability of dying at the beginning of period t from p_t^{old} to p_t^{new}, the expected values of the health-state indices $\tilde{q}_\tau^{\text{new}}$ after this risk then become

$$E_0[\tilde{q}_\tau^{\text{new}}] = \frac{(1-p_t^{\text{new}})}{(1-p_t^{\text{old}})}E_0[\tilde{q}_\tau^{\text{old}}] \quad \text{for } \tau \geq t \tag{24}$$

and the earlier expectations are unchanged: $E_0[\tilde{q}_\tau^{\text{new}}] = E_0[\tilde{q}_\tau^{\text{old}}]$ for $\tau < t$. From Equation (20), we find that the consumption adjustment associated with this change is

$$c_\tau^{\text{adj}} = \begin{cases} -\rho_\tau \ln\left(\left(\frac{(1-p_t^{\text{new}})}{(1-p_t^{\text{old}})}\right)^{R_t/R_0}\right), & \tau < t, \\ -\rho_\tau \ln\left(\left(\frac{(1-p_t^{\text{new}})}{(1-p_t^{\text{old}})}\right)^{(R_t/R_0)-1}\right), & \tau \geq t. \end{cases} \tag{25}$$

Examining this formula, we see that, as one would expect, an increase in the death risk in period t ($p_t^{\text{new}} > p_t^{\text{old}}$), leads to an increase in consumption before the risk and a decrease afterwards.

We can write an analytic expression for the derived utility associated with changes in income and death risks. Suppose that the DM starts with income and health streams $\tilde{\mathbf{x}}^{\text{old}}$ and $\tilde{\mathbf{q}}^{\text{old}}$ and the death risk in period t changes from p_t^{old} to p_t^{new} (with the new health-index stream being $\tilde{\mathbf{q}}^{\text{new}}$) and the DM's initial wealth (or period 0 income) increases by w_0. If the DM optimally adjusts consumption as specified by Equations (12) and (25), substituting these adjustments into the utility function (7), we find a derived utility of

$$\mathcal{U}(\tilde{\mathbf{x}}^{\text{old}} + w_0, \tilde{\mathbf{q}}^{\text{new}})$$
$$= a^{\text{old}} - \left(\frac{(p_t^{\text{new}} - p_t^{\text{old}})}{(1-p_t^{\text{old}})}\right)a_t^{\text{part}}$$
$$- \left(\frac{(1-p_t^{\text{new}})}{(1-p_t^{\text{old}})}\right)^{R_t/R_0} b^{\text{old}}\exp(-w_0/R_0), \tag{26}$$

where a^{old} and b^{old} are defined and interpreted as in Equations (14) and (15) (with $\tilde{q}_\tau^{\text{old}}$ in place of $\tilde{q}_t$) and

$$a_t^{\text{part}} \equiv \left(\sum_{\tau=t}^{T}k_\tau\alpha_\tau E_0[\tilde{q}_\tau^{\text{old}}]\right). \tag{27}$$

Thus, we see that the derived utility has the form of an exponential utility function, $a - b\exp(-w_0/R_0)$, where the change in death risks changes the values of a and b. The first two terms in (26) represent the expected utility of life without wealth constraints (the H-QALY score) with a_t^{part} being the H-QALYs put at risk with this change in death risks. The multiplier preceding b^{old} in (26) is less than 1 for increases in death risks; this reflects the fact that the expected "finite wealth penalty" is diminished by the increased probability of death and incorporates the DM's ability to adjust consumption to minimize this penalty in response to this change in death risks.

We can use this derived utility function to determine how much the DM should pay to reduce death risks and/or how much he or she must be paid to accept increased death risks. To illustrate, suppose that the DM starts with income and health streams $\tilde{\mathbf{x}}^{\text{old}}$ and $\tilde{\mathbf{q}}^{\text{old}}$ and the death risk in period t is changed from p_t^{old} to p_t^{new} with the new health-index stream being $\tilde{\mathbf{q}}^{\text{new}}$. What change of current wealth w_0 accompanying this change in death risk would leave the DM exactly as well off as in the status quo? Equating derived utilities, w_0 is the solution of

$$\mathcal{U}(\tilde{\mathbf{x}}^{\text{old}} - w_0, \tilde{\mathbf{q}}^{\text{new}}) = \mathcal{U}(\tilde{\mathbf{x}}^{\text{old}}, \tilde{\mathbf{q}}^{\text{old}}). \tag{28}$$

Using Equation (26) for the derived utility, we can solve explicitly for w_0 and find

$$w_0 = R_0\ln\left[\left(\frac{(1-p_t^{\text{new}})}{(1-p_t^{\text{old}})}\right)^{-R_t/R_0}\frac{1}{b^{\text{old}}}\right.$$
$$\left. \cdot \left[-\left(\frac{(p_t^{\text{new}} - p_t^{\text{old}})}{(1-p_t^{\text{old}})}\right)a_t^{\text{part}} + b^{\text{old}}\right]\right]. \tag{29}$$

If the change in risk is an improvement ($p_t^{\text{old}} > p_t^{\text{new}}$), w_0 is positive and can be interpreted as the maximum amount the DM should be willing to pay for this reduction in risk; this is the WTP for this change in risk. If the change in risk is not an improvement (e.g., $p_t^{\text{old}} < p_t^{\text{new}}$), w_0 is negative and $-w_0$ can be interpreted as the minimum amount the DM must be paid to be willing to accept this increase in risk; this is the WTA for this change in risk.

Alternatively, we might consider the change in current wealth w_0 that would substitute for a change in death risk. This w_0 is the solution of

$$\mathcal{U}(\tilde{\mathbf{x}}^{\text{old}} + w_0, \tilde{\mathbf{q}}^{\text{old}}) = \mathcal{U}(\tilde{\mathbf{x}}^{\text{old}}, \tilde{\mathbf{q}}^{\text{new}}). \tag{30}$$

Using (26), we find

$$w_0 = -R_0 \ln\left[\frac{1}{b^{\text{old}}} \left[\left(\frac{(p_t^{\text{new}} - p_t^{\text{old}})}{(1 - p_t^{\text{old}})} \right) a_t^{\text{part}} \right. \right.$$
$$\left. \left. + \left(\frac{(1 - p_t^{\text{new}})}{(1 - p_t^{\text{old}})} \right)^{R_t/R_0} b^{\text{old}} \right] \right]. \tag{31}$$

If the change in risk is an improvement (i.e., $p_t^{\text{old}} > p_t^{\text{new}}$), w_0 is positive and can be interpreted as the minimum amount the DM should be willing to accept in lieu of this reduction in risk. If the change in risk is not an improvement (e.g., $p_t^{\text{old}} < p_t^{\text{new}}$), w_0 is negative and $-w_0$ can be interpreted as the maximum amount the DM is willing to pay to avoid or prevent this increase in risk.[3]
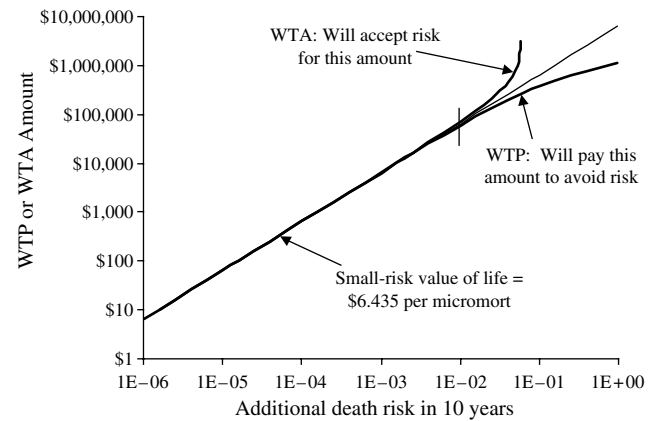
While the values given by (29) and (31) are generally different, the two coincide in the limit as the change in death risks becomes small. If we take a Taylor series expansion of (29) or (31) in p_t^{new} about $\Delta p_t = (p_t^{\text{new}} - p_t^{\text{old}}) = 0$ in the limit as Δp_t approaches 0, the values given by (29) and (31) both approach $\Delta p_t V_t$, where

$$V_t = Q_0 \left[\frac{1}{(1 - p_t^{\text{old}})} \left(a_t^{\text{part}} - \frac{R_t}{R_0} b^{\text{old}} \right) \right] \tag{32}$$

and Q_0 is the marginal value of an L-QALY given by Equation (22). We can interpret V_t as the "small-risk value of life" (sometimes called the "value of a statistical life") because small changes in death risks are valued as $\Delta p_t V_t$, which is the expected value of a Δp_t chance of losing or gaining V_t. Examining (32), we see that V_t is the marginal value of an L-QALY (Q_0) times the per unit change in expected utility or L-QALY score associated with small changes in death risks (in square brackets).

Figure 9 shows Jack's current WTP and WTA to prevent or accept additional death risks in 10 years. For

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[3] In the economics literature, the values given by Equations (29) and (31) are referred to as compensating and equivalent variations, respectively.

**Figure 9   WTP and WTA Amounts for Death Risks in 10 Years**

example, at the point marked with a vertical line in the figure, Jack should be willing to pay up to \$59,498 today to avoid a 1-in-100 chance of death in 10 years and be willing to accept such a risk for a current payment of \$71,001. Here we see that the WTP and WTA values are close for small probabilities but diverge for large probabilities. Jack's small-risk value of life for death risks in 10 years is \$6.435 million. To emphasize that this value applies only for small risks, like Howard (1980), we prefer to think of this small-risk value of life as \$6.435 per one-in-one-million probability of death or "micromort." This small-risk value is the product of a marginal value of an L-QALY (Q_0) of \$195,878 and 32.84 L-QALYs put at risk with the change in death risk in year 10. The small-risk approximation of WTPs and WTAs for death risks is fairly accurate up to risks of about 1,000 micromorts: Jack should be willing to take on an additional 1-in-1,000 risk in year 10 for \$6,494 and pay \$6,381 to avoid such a risk, compared to a value of \$6,435 given by the small risk approximation. Beyond this amount, the WTP and WTA amounts diverge. Jack's WTP to avoid death risks is bounded above by approximately \$1.1 million: Jack would not be willing to pay more than this amount to eliminate a death risk in year 10, regardless of how large, because a payment this large would leave him without the ability to enjoy a consumption stream that is preferred to certain death in 10 years. In contrast, Jack's WTA amount goes to infinity for probabilities greater than 0.057: There is no payment that would make Jack willing to accept a death risk larger than this in year 10 because consumption gains cannot offset the loss in expected utility associated with such a risk.

The WTP and WTA values for risks in different years have a similar form (they are linear for small probabilities and diverge for larger probabilities) but the values decrease with age. Figure 10 shows how Jack's small-risk value of life varies as a function of Jack's age at the time of the risk; Jack is currently

Figure 10 **Current Prices for Death Risks at Various Ages**
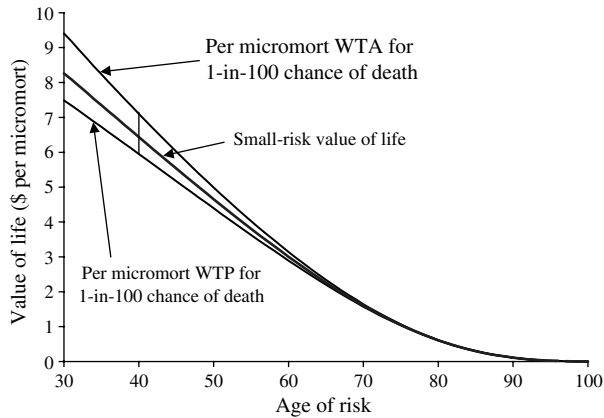
30 years old and the risk in 10 years considered in Figure 9 occurs when Jack is 40 and is noted with a cross in Figure 7. Figure 7 also shows the per micromort WTA and WTP amounts for a 1-in-100 risk of death. As one would expect, these current WTAs and WTPs decrease the later the risk occurs because one may already be dead by that time and, if alive, less of life is at stake. These WTAs and WTPs do not reflect the time value of money, as the WTP and WTA amounts are current payments for future death risks.

The amounts shown in Figures 9 and 10 are Jack's current WTP and WTA values for future risks. Figure 11 shows Jack's future small-risk value of life for an immediate death risk at different ages in different income scenarios. For example, Jack's small-risk value of life is currently (at age 30) equal to $8.263 per micromort. If Jack gets tenure at age 40, this small-risk value of life increases to $16.70 per micromort; if he is denied tenure the amount decreases to $5.02 per micromort. These dramatic changes reflect the sensitivity in the marginal value of an L-QALY to changes in wealth shown in Figure 8. The decreasing trend in the small-risk value of life between these changes in income expectations are the product of

Figure 11 **Small-Risk Value of Life as a Function of Age for Different Income Scenarios**
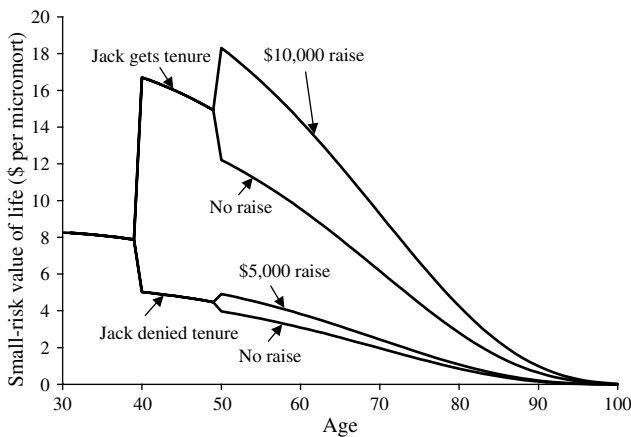
a marginal value of a QALY that is increasing at rate r (as discussed in the previous subsection) and a decreasing trend in the number of L-QALYs put at risk with future death risks; because the expected remaining L-QALY score is decreasing over time at a rate of more than $r = 2\%$ per year, the product of these two factors is a decreasing trend.

Despite this decreasing trend between changes in income expectation, Jack's expected small-risk value of life is not decreasing over time. Because the tenure and nontenure scenarios are equally likely, Jack's expected small-risk value of life at 40 is $10.85 per micromort, 31% more than his current small-risk value of life of $8.26 per micromort. Jack's expected small-risk value of life at age 57 ($8.43 per micromort) is slightly more than his small-risk value at 30. For those used to thinking in terms of H-QALYs, this is perhaps surprising as Jack's expected H-QALY score at age 30 (44.1) is more than twice his expected remaining H-QALYs at age 57 (21.4, if he lives that long). It is striking that this relatively modest uncertainty about future income can lead to such substantial increases in Jack's expected small-risk value of life over time. This can be viewed as a result of the strong convexity in the marginal value of an L-QALY as a function of wealth, as shown in Figure 7.

4.3. Valuing Changes in Expected Health States

We can value changes in health states in much the same way that we valued changes in death risks. Recall in §3.3 that when we reduced the health-state index in period t by a factor γ_t, we multiplied the expected period-t health-state index by $(1 - \gamma_t)$ in Equation (19). We can substitute the optimal consumption adjustments given by Equation (20) into the utility function (7) to find a derived utility function analogous to that for death risk (26) which we can use to determine WTPs and WTAs for changes in the health states. Using definitions analogous to those leading to (29) and (31) for death risks, we find that the DM should be willing to pay up to

$$w_0^{\text{pay}} = R_0 \ln\bigg(\frac{1}{b^{\text{old}}} \big[\gamma_t k_t \alpha_t \mathrm{E}_0[\tilde{q}_t]$$

$$+ ((1 - \gamma_t)^{[\rho_t/(1+r)^t]/R_0}) b^{\text{old}} \big] \bigg) \quad (33)$$

to avoid decreasing the expected period-t health index by a factor of γ_t and should be willing to accept such a decrease for

$$w_0^{\text{acc}} = -R_0 \ln\bigg(\frac{1}{b^{\text{old}}} \big[(1 - \gamma_t)^{[\rho_t/(1+r)^t]/R_0}$$

$$\cdot \big[-\gamma_t k_t \alpha_t \mathrm{E}_0[\tilde{q}_t] + b^{\text{old}} \big] \bigg). \quad (34)$$

As with death risks, taking a Taylor series approximation of these expressions about $\gamma_t = 0$, we find that

for small γ_t, the WTP and WTA values both converge to $\gamma_t W_t$, where

$$W_t = Q_0 \left[k_t \alpha_t \mathrm{E}_0[\tilde{q}_t] - \left(\frac{\rho_t/(1+r)^t}{R_0} \right) b^{\mathrm{old}} \right], \qquad (35)$$

where Q_0 is the marginal value of an L-QALY from Equation (22). W_t is thus the product of the marginal value of an L-QALY times the per unit reduction in L-QALYs (in the square brackets) caused by a small proportional reduction in the health-state index. Comparing (35) and (32), we note that the small-risk value of a life (V_t) for a death risk in period t can be written in terms of the sum of the small-loss values for reduction of health (W_t) for subsequent periods:

$$V_t = \frac{1}{(1-p_t^{\mathrm{old}})} \sum_{\tau=t}^{T} W_\tau. \qquad (36)$$

Intuitively, a small additional death risk in period t is equivalent to reducing the expected health state in all subsequent periods by the change in death risks.[4]

Figure 12 shows the WTP and WTA amounts for various proportional reductions in health quality occurring for one year, 10 years from now. For small losses, Jack's WTP and WTA are valued at a rate of $1,832 per 1% reduction in the quality of life for that year. For larger losses, the WTA and WTP diverge as they did with death risks. For losses approaching 100% of the year, we see a curious decline in the WTA and WTP values. In the limit as γ_t approaches 1, the optimal consumption adjustment (20) calls for consuming a negative infinite amount in period t and a positive infinite amount in other periods. In this limit, the utility function does not care about consumption in period t and, by consuming a large negative amount in this period, we can increase consumption for the rest of our life. Clearly, this extreme scenario stretches the realism of the model.

Figure 13 shows how the current small-loss value of reductions in health quality changes with the age of the loss; it also shows the WTP and WTA rates for a 25% reduction in health quality. The values initially decline rather slowly from the current value of $1,859 for a 1% reduction in health quality. The decline in values primarily reflects the probability of dying before the age is reached (i.e., the decline in the $\mathrm{E}_0[\tilde{q}_t]$ term in Equation (35)) and does not reflect the time value of money, as the WTP and WTA amounts are current payments for future reductions in health quality.

5. Two Medical Decision Problems

To illustrate the use of the model in medical decision making, we consider two simple examples involving

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[4] The $1/(1-p_t^{\mathrm{old}})$ term in (36) reflects the way changes in death risks are defined: a change of Δp_t in the period-t death risk corresponds to a proportional change of $\Delta p_t/(1-p_t^{\mathrm{old}})$; see Equation (23).

Figure 12   WTP and WTA Amounts for Reduced Health Quality
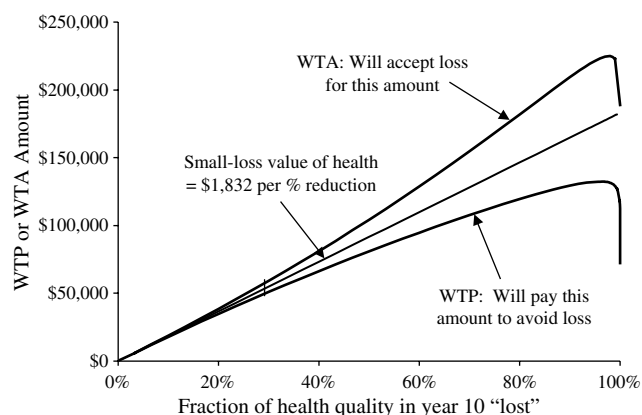
Jack. The first illustrates a full L-QALY analysis of a high-stakes medical decision problem and compares it to an H-QALY analysis. The second illustrates use of the small-risk approximations in a lower-stakes medical decision problem.

5.1. Aneurysm Example
Suppose that Jack was denied tenure and at 41 is diagnosed with a cerebral aneurysm; he must decide whether to undergo a risky surgical procedure to clip the aneurysm or to leave it untreated and risk having it rupture later. A highly simplified model of this problem is shown in the decision tree of Figure 14. If Jack has surgery, there are three immediate possible outcomes: Successful surgery eliminates the risk of future rupture. The surgery could lead to a serious disability which reduces Jack's health-state index from 1.0 to 0.5 for the remainder of his life; in this scenario he must spend an additional $5,000 per year on health maintenance for the rest of his life and has no chance of promotion. The surgery could also lead to immediate death. If Jack chooses not to have surgery, the aneurysm may rupture and cause death or a serious disability; we simplify by assuming that

Figure 13   The Current Value of Reductions in Health Quality at Different Ages
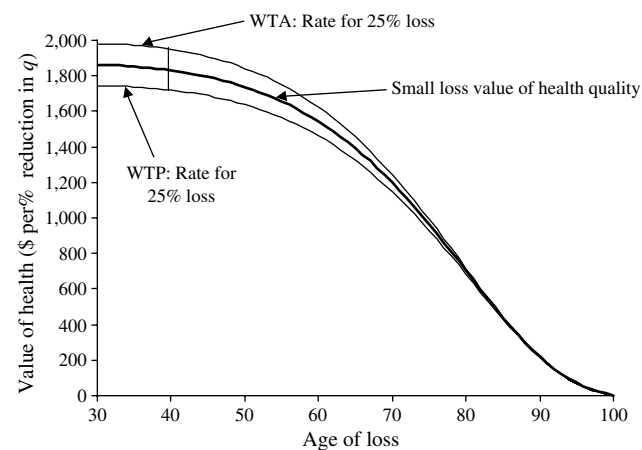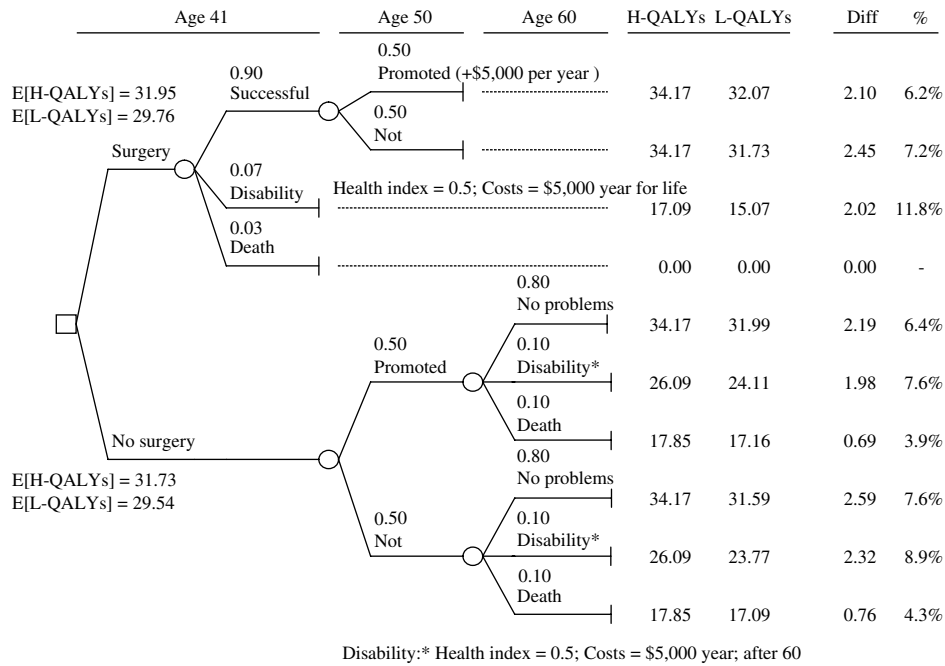
Figure 14   Decision Tree for the Aneurysm Example

			H-QALYs	L-QALYs		Diff	%
Age 41	Age 50	Age 60					
	0.50 Promoted (+$5,000 per year)		34.17	32.07		2.10	6.2%
0.90 Successful	0.50 Not		34.17	31.73		2.45	7.2%
0.07 Disability	Health index = 0.5; Costs = $5,000 year for life		17.09	15.07		2.02	11.8%
0.03 Death			0.00	0.00		0.00	–
		0.80 No problems	34.17	31.99		2.19	6.4%
	0.50 Promoted	0.10 Disability*	26.09	24.11		1.98	7.6%
		0.10 Death	17.85	17.16		0.69	3.9%
		0.80 No problems	34.17	31.59		2.59	7.6%
	0.50 Not	0.10 Disability*	26.09	23.77		2.32	8.9%
		0.10 Death	17.85	17.09		0.76	4.3%

Surgery E[H-QALYs] = 31.95 E[L-QALYs] = 29.76

No surgery E[H-QALYs] = 31.73 E[L-QALYs] = 29.54

Disability:* Health index = 0.5; Costs = $5,000 year; after 60

this risk occurs when Jack is 60. The probabilities for these risks are shown in Figure 14, as are the H-QALY and L-QALY scores for each scenario. (We assume that the risks associated with the aneurysm are in addition to standard mortality risks.) The probabilities and health-index assumptions are loosely based on Johnston et al. (1999).

Comparing the H-QALYs and L-QALYs in Figure 14 we see that in each scenario, the L-QALY score is less than the corresponding H-QALY score with percentage differences ranging from 4%–12%. These differences are "finite wealth penalties" and depend on Jack's income as well as the health outcome. The L-QALY scores depend on the time the uncertainties are resolved as well as the realized income and health-index streams. For example, in the top branch Jack is promoted and experiences a health state of 1.0 until death. The same is true on the top branch following the No Surgery decision; yet the L-QALY score is lower in the second case. This difference reflects the difference in the timing of resolution of the uncertainties. In the no surgery scenario, Jack lives with the looming risk of the aneurysm rupturing at age 60 and, as shown in Figure 15, this leads him to increase consumption before age 60. Subsequently, if he winds up surviving past 60, he must consume less in the remaining years. The lack of knowledge thus results in an ex post suboptimal allocation of consumption and a "delay premium" of the form discussed in §3.2.

Overall, the analysis shows that the expected L-QALY scores are close for the two alternatives, with the surgery score (29.76) being slightly higher than the expected score without surgery (29.54). In monetary terms, we find that Jack should be willing to pay up to $35,141 for surgery, meaning the expected utility associated with having surgery and paying an additional $35,141 for the procedure yields the same expected utility as the no surgery alternative. Jack should be willing to pay up to $234,657 for a "magical" cure for the aneurysm that eliminated the future risks without imposing the risks of the surgical procedure; this payment equates the derived utility with no aneurysm risks with the utility given by the surgical procedure.

5.2. Knee Surgery Example

To illustrate the use of the small-risk approximation, we consider another simple example involving Jack. Suppose that Jack is 30 and has injured his knee and is

Figure 15   Consumption Streams in the Aneurysm Example
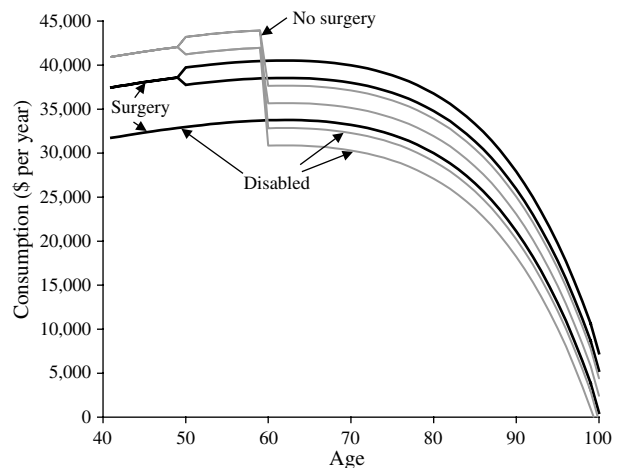
considering surgery to repair it. Surgery is not necessary, but, without it, Jack will occasionally experience pain and will be unable to participate in certain activities (running, skiing, etc.) that he enjoys. He estimates that this condition, if untreated, will reduce his future health quality (q_t) by 1.5% per year for the rest of his life. If he has surgery, there is a 0.15 probability that surgery will be unsuccessful and he will be slightly worse off after the surgery than before and experience a 2% per year reduction in health quality. There is also a 0.0001 probability of immediate death from complications of surgery. Finally, there is a 0.8499 probability that the procedure will be successful and will restore him to full health. Surgery will cost Jack $15,000 (it is not covered by his insurance) and will require him to take a medical leave and forgo $5,000 in salary. In addition, recovery from surgery will result in a 5% reduction in health quality in the current year. Should Jack have surgery?

The stakes are significant in this example, but small enough to allow us to treat the problem using the small-risk approximations. This approximate analysis is conducted in monetary units and is displayed in Table 1. The direct costs of surgery and forgone salary total $20,000. The cost of recovery is a 5% reduction of health quality for one year and may be approximated as 5% of the current small-loss value of health, which as discussed in §4.3 is $1,859 per 1% reduction in health; the cost of recovery is thus approximately $9,294. The long-term health effects will be measured from a reference point of perfect health ($q_t = 1$) and appear as costs in Table 1. The current small-risk value of death was given in §4.2 as $8.263 per micromort or $8,262,665 per unit of probability. The long-term health effects are assumed to be permanent and can thus be approximated as percentages of this small-risk value of life. The equivalent cost of a 1.5% reduction in health quality without surgery is 1.5% × $8,262,665 = $123,940. The total (expected) equivalent cost with surgery is $54,786, so the net benefit of surgery is $69,305.

We can conduct the same approximate analysis in L-QALY terms and find that surgery's expected

L-QALY score is 41.896 and no surgery is 41.544. Given the marginal value of an L-QALY of $195,878 per L-QALY, the difference 0.352 in L-QALY scores is equivalent to the difference in expected costs of $69,305. The exact L-QALY scores differ from these approximate ones by less than 0.001.

6. Conclusions and Discussion

The primary contribution of this paper is the development and analysis of a tractable model that allows one to study tradeoffs between income and health risks under uncertainty. The model incorporates QALY-like preferences for health states and takes into account the DM's ability to adjust consumption in response to changes in expectations about future health status and income. We can use the model to analyze major life decisions (e.g., should I have surgery? should I retire?) in much the same way that H-QALY models are currently used. The small-risk values of life and small-loss value of health generated by the model allow one to easily evaluate small stakes decisions requiring tradeoffs between financial and health outcomes, as illustrated in the example in §5.2. While our focus has been on individual decision making, the model can also be used for cost-effectiveness analysis and cost-benefit analysis. In a cost-effectiveness analysis, instead of considering the ratios of H-QALYs gained per dollar spent, one might consider L-QALYs gained per dollar spent and better capture the financial implications of health interventions. In a cost-benefit analysis, one might measure the total value of some policy or intervention by considering the sum of the WTPs given by this model for the affected population.

The main difference between our model and the standard H-QALY model is its ability to integrate financial tradeoffs and consumption decisions into the analysis. This leads to some qualitative differences in the evaluations. In the standard H-QALY model of the form of Equation (1), all health-state indices are weighted equally. Many have argued that future health indices in H-QALY models should be discounted at the same rate used to discount costs (see

Table 1 Costs and Benefits of Jack's Surgery Decision

Probability	Scenario	Direct costs ($)	Recovery ($)	Long-term health costs ($)	Total cost ($)	Contribution to expected cost ($)
0.8499	Successful surgery (no reduction in health)	20,000	9,294	0	29,294	24,897
0.1500	Not successful (2% reduction in health)	20,000	9,294	165,253	194,547	29,182
0.0001	Surgical death (100% reduction in health)			8,262,665	8,262,665	826
	Expected total cost					54,905
	No surgery (1.5% reduction in health)	0	0	123,940	123,940	

Gold et al. 1996 for discussion). In our model "discounting" of health indices is endogenously determined by the optimal consumption levels as the health indices are weighted by the consumption utilities $u_t(c_t)$. The resulting pattern of discounting does not fit that given by any constant discount rate. For example, examining the optimal consumption streams of Figures 4 and 15, we see that the optimal consumption strategies call for increasing consumption over the early years, which leads to increasing weights on the early health indices. However, health-state indices in later years receive less weight because the optimal consumption strategies call for lower consumption levels in later years, recognizing that one may not live that long. Howard's model (Howard 1980, 1984) considers tradeoffs between financial and health objectives, but, like the standard H-QALY model, it weighs all health indices equally as it assumes that the overall utility is a function of the DM's constant annual consumption and the H-QALY score.

There are many other topics that should be explored in future research. First, we need to address assessment issues. We believe that the health-state indices used in H-QALY studies can also be used to specify health-state indices in our framework. For example, Torrance and coauthors (Torrance et al. 1982, Furlong et al. 2001) have developed a health utilities index that is used in many H-QALY studies. However, we would need to verify the use of such utilities as a multiplier on consumption utilities as they are used in this framework. The other utility parameters can be assessed using standard utility assessment techniques (e.g., Keeney and Raiffa 1976). Alternatively, one might develop new techniques specifically for this application. In either case, given the sensitivity of the results to these parameters, we recommend using consistency checks in the assessment process. Furthermore, the application of the model should include thorough sensitivity analyses and careful scrutiny of the results.

The additive-exponential utility function that we have used here is perhaps best viewed as approximation of more realistic utility functions. While this functional form greatly simplifies computation, it would be useful to the study other utility functions of the form of (2) with a different or general consumption utility function. Alternatively, one might study forms that are additive across periods but do not assume that consumption and health-state index are multiplicative within each period. While we would expect many of the same qualitative properties of the additive-exponential model (e.g., WTAs and WTPs are equal for small risks) to hold for more general models, we do not expect the consumption optimization problem to decompose as it does here. We would also like to better understand the implications of imposing borrowing and consumption constraints

and bequest utilities. Given the computational benefits of the simple unconstrained additive-exponential utility, we should also study how this form approximates results for more complex utility functions and financial models.

Perhaps the best way to address the assessment and approximation questions and understand the difference in results between L-QALY and H-QALY-based analyses is to apply the model developed here to problems that have already been studied carefully using H-QALY models. For example, Weinstein and coauthors used a Markov model of coronary heart disease to study the value of exercise, cholesterol treatments, and/or smoking cessation in a QALY framework (see e.g., Weinstein et al. 1987, Prosser et al. 2000). Alternatively, we might consider the full dynamic model of aneurysm treatment decisions in Johnston et al. (1999). Applying our model in these problems, we could examine the full cost and benefits and study the DM's WTP for such interventions. We might find, for example, that dietary changes to lower cholesterol are worth, say, $500 or $50,000 to the DM. In the former case, the DM may reach the conclusion that such an intervention is not worthwhile (I won't give up eating hamburgers for $500!). In the latter case, however, the DM might understand the value of the intervention and have the conviction to maintain the dietary regimen. Considering these kinds of examples should help us to better understand the computational and assessment burdens associated with the model as well the potential benefits associated with its use.

Appendix 1. Deriving the First-Order Conditions
To derive the first-order conditions (5), we write the optimization problem (4) in a recursive dynamic programming form. Let $\tilde{\mathbf{x}}_t = (\tilde{x}_t, \ldots, \tilde{x}_T)$ and health-index stream $\tilde{\mathbf{q}}_t = (\tilde{q}_t, \ldots, \tilde{q}_T)$ denote the income and health-index streams beginning time t and let $\tilde{\mathbf{x}}_t + w_t$ denote the income stream $(\tilde{x}_t + w_t, \ldots, \tilde{x}_T)$ that adds an initial sum w_t to the first period; this w_t will represent accumulated wealth or savings available at time t. We let $\mathcal{U}_t(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t)$ denote a derived utility function that includes only period utilities after period t. The DM's consumption optimization problem can then be written as

$$\mathcal{U}_t(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t)$$

$$= \max_{c_t}\{q_t u_t(c_t) + \mathrm{E}_t[\mathcal{U}_{t+1}(\tilde{\mathbf{x}}_{t+1} + (w_t + x_t - c_t)(1+r), \tilde{\mathbf{q}}_{t+1})]\}$$

$$\text{for } t < T \quad \text{and} \quad (A1)$$

$$\mathcal{U}_T(\tilde{\mathbf{x}}_T + w_T, \tilde{\mathbf{q}}_T) = q_T u_T(x_T + w_T). \quad (A2)$$

Here the terminal condition (A2) captures the requirement that the DM must have zero savings at time T. The

first-order conditions for the period-t maximization problem (A1) (differentiating with respect to c_t and setting equal to zero) imply that the optimal consumption choice c_t^* satisfies

$$q_t u_t'(c_t^*) = (1+r)E_t[\mathcal{U}_{t+1}'(\tilde{\mathbf{x}}_{t+1} + (w_t + x_t - c_t^*)(1+r), \tilde{\mathbf{q}}_{t+1})], \tag{A3}$$

where u_t' is the derivative of the consumption utility and $\mathcal{U}_t'$ is the partial derivative of $\mathcal{U}_t(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t)$ with respect to w_t. Thus, the marginal utility associated period-t consumption is selected to match the expected marginal utility of period-t wealth. Now let us calculate $\mathcal{U}_t'$ directly, using the chain rule:

$$\mathcal{U}_t'(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t)$$
$$= \frac{\partial \mathcal{U}}{\partial w_t}$$
$$= \{q_t u_t'(c_t^*)$$
$$\quad - (1+r)E_t[\mathcal{U}_{t+1}'(\tilde{\mathbf{x}}_{t+1} + (w_t + x_t - c_t^*)(1+r), \tilde{\mathbf{q}}_{t+1})]\} \frac{\partial c_t^*}{\partial w_t}$$
$$\quad + (1+r)E_t[\mathcal{U}_{t+1}'(\tilde{\mathbf{x}}_{t+1} + (w_t + x_t - c_t^*)(1+r), \tilde{\mathbf{q}}_{t+1})].$$

The bracketed ({ }) term here is zero by (A3); therefore, taking $w_{t+1}^* = (w_t + x_t - c_t^*)(1+r)$, we have

$$\mathcal{U}_t'(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t) = (1+r)E_t[\mathcal{U}_{t+1}'(\tilde{\mathbf{x}}_{t+1} + w_{t+1}^*, \tilde{\mathbf{q}}_{t+1})]. \tag{A4}$$

This with (A3) implies

$$\mathcal{U}_t'(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t) = q_t u_t'(c_t^*). \tag{A5}$$

Applying (A4) recursively, we have

$$(1+r)^t \mathcal{U}_t'(\tilde{\mathbf{x}}_t + w_t, \tilde{\mathbf{q}}_t) = (1+r)^\tau E_t[\mathcal{U}_\tau'(\tilde{\mathbf{x}}_\tau + w_\tau^*, \tilde{\mathbf{q}}_\tau)] \tag{A6}$$

for all t and $\tau > t$. Substituting (A5) into (A6), we arrive at (5). Equation (A6) also shows that, absent changes in expectations about future income or health-index streams ($\tilde{\mathbf{x}}_t, \tilde{\mathbf{q}}_t$), marginal utility $\mathcal{U}_t'$ decreases at the interest rate r, or, equivalently as discussed in §4.1, the marginal value of an L-QALY given as $1/\mathcal{U}_t'$, increases at the interest rate r.

Appendix 2. Extending Smith's (1998) Procedure for Valuing Income Streams

In this appendix, we discuss the use of the results of Smith (1998) to evaluate income streams. That paper did not consider uncertainty about health states and considered utility functions of the form

$$\sum_{t=0}^{T} k_t \exp(-c_t/\rho_t) \tag{A7}$$

rather than the form of Equation (7). However, if we assume that the new income stream $\tilde{\mathbf{x}}^{adj}$ to be evaluated is independent of the health-index stream $\tilde{\mathbf{q}}$ and prior income stream $\tilde{\mathbf{x}}^{old}$, the more complex form (7) can be reduced to the form of Equation (A7) for the purposes of evaluating $\tilde{\mathbf{x}}^{adj}$. Suppose that we write $\tilde{\mathbf{c}}^{new} = \tilde{\mathbf{c}}^{old} + \mathbf{c}^{adj}$ and consider the expected utility with the utility model (7):

$$E[U(\tilde{\mathbf{c}}^{old} + \mathbf{c}^{adj}, \tilde{\mathbf{q}})] = \sum_{t=0}^{T} k_t \alpha_t E[\tilde{q}_t]$$
$$\quad - \sum_{t=0}^{T} (k_t E[\tilde{q}_t \exp(-\tilde{\mathbf{c}}^{old}/\rho_t)]) \cdot E[\exp(-\tilde{\mathbf{c}}^{adj}/\rho_t)]. \tag{A8}$$

Here, we have used the independence assumption to factor the expectations in the second summation: independence in the income streams $\tilde{\mathbf{x}}^{old}$ and $\tilde{\mathbf{x}}^{adj}$ lead to independent consumption streams $\tilde{\mathbf{c}}^{old}$ and $\mathbf{c}^{adj}$. When evaluating the incremental income stream $\tilde{\mathbf{x}}^{adj}$, the first term in (A8) is simply a constant and can be dropped. If we take the k_ts in (A7) to be the $(k_t E[\tilde{q}_t \exp(-\tilde{\mathbf{c}}^{old}/\rho_t)])$ terms from (A8), we see that the expected utility in (A8) can be reduced to the consideration of the utility function (A7). Thus, we can use the procedures and results of Smith (1998) to evaluate independent, incremental income streams in the more complex setting of this paper.

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