Decision Analysis
Vol. 2, No. 2, June 2005, pp. 103–109
ISSN 1545-8490 | EISSN 1545-8504 | 05 | 0202 | 0103

DOI 10.1287/deca.1050.0042

Response to Comments on Brandão et al. (2005)

Luiz E. Brandão
IAG Business School, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ 22453-900, Brazil, brandao@iag.puc-rio.br

James S. Dyer, Warren J. Hahn
McCombs School of Business, The University of Texas at Austin, Austin, Texas 78712
{jim.dyer@mccombs.utexas.edu, warren.hahn@phd.mccombs.utexas.edu}

In this note, we respond to Smith's (2005) discussion of the approach outlined in our paper (Brandão et al. 2005) on using traditional decision analysis methods to solve real-options problems. Our response addresses several areas where we largely agree with Smith, but have different views on modeling preferences or on the practicality of implementing alternative modeling approaches. We view the issue raised by Smith on the estimation of process volatility to be a valid concern and propose a modification to our method to address this problem.

Key words: decision analysis; decision trees; real options
History: Received on August 5, 2005. Accepted by Robert Clemen and Don Kleinmuntz on September 9, 2005, after 2 revisions.

1. Introduction

Smith (2005) provides a discussion of our proposed approach (Brandão et al. 2005; hereafter, BDH) for using traditional decision analysis methods to solve real-options problems. In addition, he provides an overview of some alternative approaches to solving real-options problems, and a discussion of their relative advantages. We consider this dialogue to be a constructive one, and have a few observations and clarifications to add regarding his comments.

In his discussion, Smith identifies a number of areas where he disagrees with our approach. We feel that it is appropriate to point out where Smith's concerns are based on modeling preferences rather than on fundamental limitations of the BDH approach. We also point out the areas where we agree with Smith in principle, but have different views on the practicality of the alternative modeling approaches he discusses. Finally, Smith raises a substantive point regarding the estimation of the project volatility that we use in modeling the stochastic process associated with the underlying value of the project. We consider this criticism to be a valid one, and we suggest a revised approach to calculating the volatility of project value as a result.

Smith begins his discussion by describing the BDH approach as the three-step process that we used to illustrate the use of decision trees to approximate the uncertainty in project cash flows over time with a geometric Brownian motion (GBM). While his description of this example is an accurate one, we view the BDH approach as being much more general, as indicated by our discussion of alternative ways that it could be implemented, including some illustrated by Smith.

The "BDH approach" is based on the use of event trees to approximate an appropriate stochastic process using risk-neutral valuation by calculating a net present value (NPV) with risk-neutral probabilities and discounting at the risk-free rate, as Smith notes. Within this structure it is possible to solve some real-options problems using commercially available decision-tree software by simply introducing these options as decision nodes. This idea was first illustrated with a simple example involving the approximation of the uncertainty in project value over time with a simple GBM model. The second illustration was based on the work by Copeland and Antikarov (2001), and is the focus of Smith's comments.

This approach can be carried out in three steps as follows:

(1) Calculate the expected NPV of the project without options. In our example, we used a traditional discounted cash flow analysis using a risk-adjusted

discount rate, but we also stated in the discussion that a risk-neutral approach to calculating the NPV should be used if the appropriate market information is available. Smith has provided a "fully risk-neutral approach" that illustrates this idea, and his example allows us to demonstrate that the BDH approach is compatible with this approach.

(2) Using this discounted cash flow model and a Monte Carlo simulation that describes the uncertainty in the cash flows, estimate the parameters of a stochastic process that is used to approximate these cash flows. If a standard GBM approximation is used, then this step requires only the estimation of the volatility of the process in any arbitrary period because the volatility of a GBM process remains constant over time. In our discussion, we also noted that other stochastic processes could be considered for this approximation, including arithmetic Brownian motion or a process with time-dependent volatilities. Smith's introduction of the fully risk-neutral example allows us to illustrate the latter alternative.

(3) Build an event tree that uses the stochastic process to approximate the uncertainty in the project value over time using risk-neutral valuation and incorporating options in this tree with decision nodes. A binomial tree would be used to approximate a GBM, but other tree structures could also be used.

Our response will follow the same structure as Smith's comments.

2. Binomial Lattices vs. Binomial Trees

Smith provides a valuable addition to our paper by solving the oil production example problem using a lattice approach rather than the binomial tree approach we employ. As we state in our paper, and as Smith now shows explicitly, either approach can be used to obtain the same result. With both approaches presented, the reader can select the one he or she would prefer to implement.

As the number of options associated with a problem increases, the logical statements in a lattice become increasingly complex and, arguably, more susceptible to errors. Panko (1998) reports on the high error rates associated with large spreadsheets, and cites audit evidence that 20% to 40% of all spreadsheets contain errors. Naturally, errors of transposition and calculation can also occur in a decision tree, but we believe that the logic of a decision tree is relatively transparent and much more open to inspection by others not skilled at developing spreadsheet formulae.

When dealing with more complex real-options problems, we have found it useful to develop our models in a binomial tree format for a relatively small number of time periods (less than 30), and to create lattices that match these results as a check of their logic. Then, it is straightforward to increase the number of time periods in a lattice in order to obtain more precision based on their computational advantages. As these lattices become larger (50 or more periods for example), they may be coded in programming languages such as Visual Basic, Matlab, or C++, rather than in spreadsheets, and again we have found it helpful to debug these programs by matching the results for a smaller number of time periods to those from a binomial tree.

We view the choice between binomial trees and lattices to be primarily a matter of modeling preference. As shown by Copeland and Antikarov (2001) and Copeland and Tufano (2004), the basic approach we discuss was actually introduced in lattice format. However, while Smith and others may prefer using lattices for these types of problems, we anticipate that many members of the decision analysis community may prefer working in the binomial tree format.

3. Estimating Cash Flow Volatilities

By duplicating the spreadsheet we created to simulate the uncertain cash flows for our example, Smith has indeed identified an issue with our use of Copeland and Antikarov's approach (hereafter CA) to estimating the volatility of the GBM approximation for the uncertainty in project value over time.

Recall that in our approach, we use simulated cash flows in the spreadsheet to calculate the period-by-period project values, from which we can calculate period-by-period project returns. If a GBM stochastic process provides a reasonable approximation to the evolution of project value, then the standard deviations of these period-by-period returns will be approximately equal. If this is the case, then we can arbitrarily use the standard deviation of the project returns in Period 1 to specify the volatility parameter of the stochastic process. Finally, this tree or lattice spins off state- and period-specific cash flows for the project and allows us to model optimal decision making and option value for the project.

Following CA (2001, Chapter 9), we defined the random variable z as the return between time 0 and time period 1 using the relationship

$$z = \ln\left(\frac{V_1}{\overline{V}_0}\right) = \ln\left(\frac{C_1 + PV_1(C_2, \ldots, C_n)}{\overline{V}_0}\right), \qquad (1)$$

where C_i is the stochastic cash flow from period i and $\overline{V}_0$ is the deterministic present value of the project at time zero. We used the estimate of the standard deviation of (1) to obtain the volatility of 46.6% for the oil production project present value.

Smith (2005) observes that our approach overestimates the volatility of cash flows. He bases his argument on the 10th and 90th percentile values observed for the simulated cash flows in the spreadsheet, as compared to the 10th and 90th percentiles that result from simulating the project values with the 46.6% volatility parameter we obtained. He observes that this standard deviation would be appropriate if the values followed a GBM with constant volatility and V_1 was Period 1's expected NPV of subsequent cash flows rather than a realization of subsequent cash flows. In contrast, in the procedure we have used, V_1 is the present value of realizations of future cash flows across all periods that are generated in the simulation and reflects the resolution of all future uncertainties.

Smith's comment suggests a modification to the specification of the simulation variable z. By changing the simulation model so that only C_1 is stochastic, and specifying $C_2, \ldots, C_n$ as expected values conditional on the outcomes of C_1, we capture only the variability in V_1 that is due to the uncertainty resolved up to that point. Thus, a better estimate for the volatility of project value can be obtained using the expression for project returns shown in Equation (2):

$$z = \ln\left(\frac{C_1 + PV_1(E_1(C_2), \ldots, E_1(C_n) \mid C_1)}{\overline{V}_0}\right). \qquad (2)$$

We have found that this specification does indeed provide a better estimate of the volatility of the GBM approximation of the project value V_0. For example, making this change to the simulation model for the oil production example, we find the standard deviation of z is reduced from 46.6% to 27.9%. This result is close to the 25.5% that Smith obtains by searching for a volatility for the GBM approximation that matches, as well as possible, the mean and standard deviation of the distribution of the simulated values of the project value V_0.

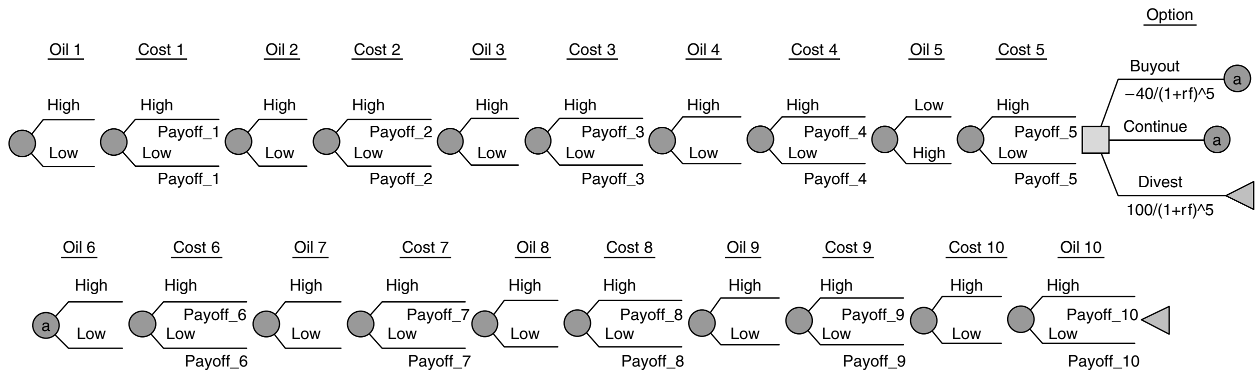
We can compare the revised approximation of the GBM for project value with the result of binomial approximations to the two individual uncertainties in this problem in a manner consistent with the Smith and Nau (1995) approach. For simplicity and convenience, we consider the fully risk-neutral version of the problem that Smith discusses in his comments. In this model, Smith assumes that the oil price process will be risk neutral if the drift rate is adjusted by subtracting a risk premium of 3%, so that the risk-neutral drift rate is 0%. Making this change and discounting cash flows using the risk-free rate yields a project NPV of \$392 million, as shown in Figure 4 of Smith's comments.

We show the DPL™ model in Figure 1 for the approach with the price and cost uncertainties explicitly modeled and with the option to expand or abandon at the end of Year 5. The "Payoff_*i*" on the branches reflect the cash flow calculations, in this case from the uncertain oil price and variable operating costs.

Solving this tree yields a project value with options of \$423.2 million. This compares to Model/Assumption 4 in Smith's comments, in which he obtained a value of \$421 million using the Longstaff and Schwartz (2001) method based on simulation combined with linear regression.

As illustrated in Figure 1, this tree becomes quite large even for this relatively simple example with only 10 periods and two uncertainties, which demonstrates the desirability of using a univariate approximation of the stochastic process. Using the BDH approach, we can solve this problem by building a tree for project value with only one binary event in each period. Simulating with the worksheet shown in Figure 4 of Smith's comments and using the revised method (2) yields a standard deviation for z of 31.8%, an estimate of the volatility. We also estimated this volatility using Smith's approach, by matching the mean and standard deviation of the distribution of the simulated values of the project present value V_0 with the same statistics for V_0 from a simulated GBM.

Figure 1 **Decision Tree for Oil Production Example**



This approach provided an estimate of 29% for the project volatility, which is fairly close to the value we obtained from the simulation of z.

We then constructed a univariate tree for the project present value assuming a GBM with volatility $\sigma = 31.8\%$ and a project value at time zero of \$392 million. Inserting the decision nodes for the options in Year 5 and solving this tree gives a value of \$418 million. This solution of \$418 million differs by less than 1% from \$421 million obtained by Smith with the simulation-based approach. This difference would be within the acceptable margin of error for many practical applications.

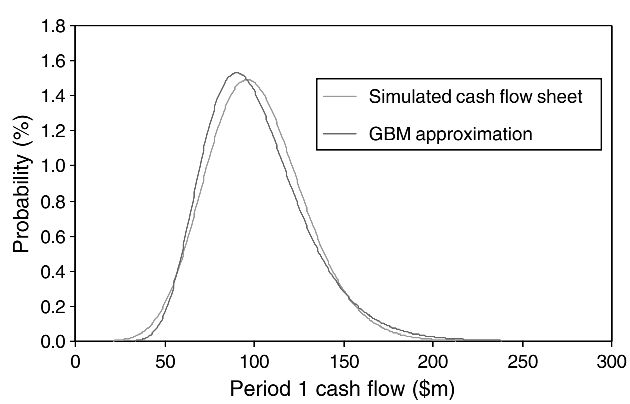
While we disagree with Smith about the usefulness of simple univariate approximations of the value process, we agree that it is important to consider how well the GBM approximation matches the uncertainty in the actual cash flows for a particular application. Two notable characteristics of a GBM stochastic process are that the values at any point in time should be lognormally distributed and that the standard deviations associated with the period-to-period returns should be constant. Both of these characteristics can be checked using the results of the simulation model.

For example, based on a Chi-square test, we cannot reject the hypothesis that the simulated values of the Period 1 present values in the original example problem are lognormally distributed using a p-value of 5%. To calculate the statistic, we used a sample size of 5,000 and 55 degrees of freedom, which is based on partitioning the data into 56 bins. The same result holds for the revised risk-neutral version of the problem.

Because the assumption of lognormally distributed present values appears to be valid, we expect to be able to specify a GBM that will reasonably approximate the distribution of cash flows. We check this for Period 1 by comparing the distribution of cash flows simulated from the cash flow spreadsheet with the distribution of cash flows generated from a simulated GBM for the present value. These two distributions are shown in Figure 2 and are similar, so we consider this GBM approximation to be a reasonable one.

We can also test the constant volatility assumption by simulating z for all periods, not just Period 1, and calculating the period-by-period volatilities. For our original example problem, the volatilities were relatively constant, ranging from 27.8% to 28.6%. However, for the fully risk-neutral version of the example problem as revised by Smith, we observed that the volatilities steadily increase from 31.8% in Period 1 to a maximum of 45.6% in Period 10. This occurs in the fully risk-neutral version of the problem because

Figure 2 **Comparison of Probability Density Functions for Cash Flows**



the growth rate for the oil prices is assumed to be zero while the growth rate for the costs is 2%. This causes the cost/revenue ratio to increase over time, creating a leverage effect on the cash flows that causes the increase in volatility. Our assumed GBM model with constant volatility of 31.8% does not capture this increase and slightly underestimates the actual overall volatility of project value in this case. This is reflected in the small downward bias in our result as compared with results from a model that explicitly includes price and cost.

Because we use a binomial tree rather than a recombining lattice, we can easily accommodate this changing volatility in our model as explained in the discussion section of our paper. Doing so results in a value with options of $420.3 million, which is within 0.17% of the result from Smith's Model/Assumption 4. The decision trees and the corresponding spreadsheets for these examples are available in the online supplements section of the *Decision Analysis* website.

The volatility in each time period in this example is not constant, and in fact it may be value dependent, but this approach provides a good approximation for this problem. While the simple GBM model was used in our examples for illustration purposes, we do not consider it to be an essential component of the "three-step BDH process" as suggested by Smith. This point was considered in some detail in the discussion section of our paper, where we describe some limitations of the GBM assumption and suggest the use of other stochastic processes to model project value when a GBM is not appropriate. This generality is illustrated with the use of different period-by-period volatilities for the fully risk-neutral model as explained above. The procedures for approximating value processes need to be sensitive to the kinds of options being considered and to the particulars of the processes.

As we also emphasized in the discussion, if the project is subject to uncertainties whose resolution may affect the project risk, such as new information obtained from geophysical surveys or initial drilling, then these uncertainties should be modeled separately at the decision-tree level rather than in the simulation model. These types of risks tend to decrease with time and may not be consistent with an effort to approximate their impacts on cash flows with a relatively simple stochastic process. However, we recognize that many projects may have cash flow streams and complex option structures that cannot be approximated closely using these ideas, and other modeling techniques may be more appropriate.

Therefore, we do agree with Smith that the problem of approximating a high-dimensional stochastic process with a low-dimensional summary process is an interesting research problem that goes well beyond the question of how to pick a volatility parameter for a GBM process. We also believe, and we think that Smith would agree, that a univariate approximation can be a useful tool for analyzing many problems with multidimensional stochastic processes when care is taken to ensure that the appropriate assumptions are reasonably satisfied.

4. On Risk-Neutral Valuation

Smith prefers to use a "fully risk-neutral" approach to solving real-option valuation problems. We do as well whenever such an approach is appropriate and computationally feasible. In §6 of our paper we state that risk-neutral forecasts for uncertain variables should be used when possible, and that such forecasts can be incorporated into the CA or BDH methods. Furthermore, in §2 of this response we have shown that the BDH approach applied with risk-neutral forecasts leads to similar values to those from the "fully risk-neutral" approach.

In our discussion we also questioned whether it would be practical in many cases to adopt this approach. As we noted, information regarding the risks of individual factors may be difficult to obtain, and some of these risks may be correlated. For example, variable operating costs associated with an oil production project may be correlated with oil prices. In such cases it may be easier to estimate risk-adjusted discount rates for projects using market data for benchmark publicly traded companies because many practitioners are more familiar with this approach and may find it more acceptable.

We are pleased to see the details Smith provides in the appendix of his comments regarding a method for including "semimarket correlated" uncertainties in the integrated Smith and Nau (1995) valuation framework. As far as we are aware this approach has not been published before.

5. On the Choice of Underlying Uncertainty

In this section, Smith argues that the BDH approach is only applicable to projects with "scale" options, and thus is not sufficiently general. We acknowledge that this approach will not be the most appropriate modeling choice for all real-options problems, although some flexibility can be accommodated in these models.

Smith also questions the use of the project value without options as the underlying uncertainty, and champions the use of simulation-based methods for evaluating high-dimensional valuation problems. We agree with Smith that simulation provides a robust method for modeling multiple uncertainties, including those that follow non-GBM stochastic processes such as mean-reverting or jump diffusion processes. In fact, the CA and BDH approaches utilize simulation methods, albeit in a different and more limited way.

Option valuation problems typically require a backward recursive solution to optimal decision policy at any point in time up until the end of the project's life. Monte Carlo simulation, however, is a "forward-looking" process, and is not naturally suited to this type of problem. As Smith notes, several different methods have been proposed for modifying simulation methods to facilitate approximation of the value function to evaluate optimal stopping decisions along simulated paths for the underlying asset. These include linear regression (Longstaff and Schwartz 2001), linear programming (de Farias and Van Roy 2003), and policy iteration (Ibanez and Zapatero 2004). Smith is correct that these methods, in principle, allow us to directly consider each individual underlying uncertainty; however, in our experience the incremental modeling and computational requirements of these approaches can be significant.

For example, we have used the Longstaff and Schwartz (2001) method to model problems with an underlying mean-reverting uncertainty and have found it to work well for a simple abandon option. However, for more realistic problems with complex and/or multiple concurrent options, it may be more challenging to program the required decision-making logic into the model, as Smith also notes. As an alternative, Gamba (2002) applies the Longstaff and Schwartz method separately to each of the individual options in a problem and then finds ways to aggregate the incremental values from each option to derive a single value for the value of the project.

The solution methods associated with these simulation models also are subject to spreadsheet programming errors, and may appear to managers and other decision makers as a "black box." We believe that these tools are important approaches to real-options valuation, but suffer from many of the same drawbacks as lattices with regard to transparency and intuition, and in terms of their ability to model relatively complex real-options problems.

6. Summary

In §§2 and 5, Smith has provided a valuable discussion of alternative approaches to solving real-options problems by describing the use of lattices and simulation for this purpose. We believe that these tools can be valuable, and that no one technique will be the best choice for all real-options problems. The choice of the appropriate modeling technique should depend on the characteristics of a specific problem, the needs of the decision maker, and the modeling preferences of the analyst. We hope that the readers will become familiar with all of these tools and make wise choices among them.

In §4, Smith argued for a "fully risk-neutral" approach to determining the present value of a risky project. We agree that this approach should be used when the necessary information is available, but we think that other considerations will justify the use of the mix of risk-adjusted and risk-neutral approaches in some cases. Once again, we view the choice between these approaches as being problem dependent.

In §3, Smith highlighted a problem with the calculation of the volatility of the project value in the BDH approach. Fortunately, his intuition also suggested a viable solution to this problem, and we believe that the modified approach described in this response addresses this issue. As we discussed in our paper, the GBM stochastic process must be a reasonable approximation to project value in order for this approach to be used. Otherwise, consideration should be given to the use of other stochastic processes to model project value or to other solution strategies such as the simulation-based approaches described by Smith.

In summary, we agree with the majority of Smith's comments, and believe that most of our apparent disagreements reflect different preferences and views on the practical implementation of the various available approaches to modeling problems. We think that this dialogue has been a valuable one in clarifying a number of issues related to the role of decision analysis in the solution of real-options problems, and hope that readers will concur. For a related discussion of alternative approaches to solving real-options problems, we also recommend Triantis (2005), Borison (2005), and Copeland and Antikarov (2005).

References

Borison, A. 2005. Real options: Where are the emperor's clothes. *J. Appl. Corporate Finance* **17**(2) 17–31.

Brandão, L., J. Dyer, W. Hahn. 2005. Using binomial trees to solve real-option valuation problems. *Decision Anal.* **2**(2) 69–88.

Copeland, T., V. Antikarov. 2001. *Real Options*. Texere LLC, New York.

Copeland, T., V. Antikarov. 2005. Real options: Meeting the Georgetown challenge. *J. Appl. Corporate Finance* **17**(2) 32–51.

Copeland, T., P. Tufano. 2004. A real-world way to manage real options. *Harvard Bus. Rev.* **82**(3) 90–99.

de Farias, D., B. Van Roy. 2003. The linear programming approach to approximate dynamic programming. *Oper. Res.* **51** 850–865.

Gamba, A. 2002. Real options valuation: A Monte Carlo approach. Working paper 2002/3, EFA 2002 Berlin Meetings, Faculty of Management, University of Calgary.

Ibanez, A., F. Zapatero. 2004. Monte Carlo valuation of American options through computation of the optimal exercise frontier. *J. Financial Quant. Anal.* **39** 253–275.

Longstaff, F., E. Schwartz. 2001. Valuing American options by simulation: A simple least-squares approach. *Rev. Financial Stud.* **14** 113–147.

Panko, R. 1998. What we know about spreadsheet errors. *J. End User Comput.* **10** 15–21.

Smith, J. 2005. Alternative approaches for solving real options problems (Comment on Brandão et al. 2005). *Decision Anal.* **2**(2) 89–102.

Smith, J., R. Nau. 1995. Valuing risky projects: Option pricing theory and decision analysis. *Management Sci.* **14**(5) 795–816.

Triantis, A. 2005. Realizing the potential of real options: Does theory meet practice. *J. Appl. Corporate Finance* **17**(2) 8–16.