Risk Sharing, Fiduciary Duty, and Corporate Risk Attitudes

James E. Smith
Fuqua School of Business, Duke University, Box 90120, Durham, North Carolina 27708-0120, jes9@mail.duke.edu

In this paper, we consider the problem of determining corporate risk preferences that reflect the preferences of the company's shareholders. Although we do not assume that the company necessarily has a utility function, we can use shareholder risk preferences to place bounds on the firm's risk tolerances and certainty equivalents. Using these bounds with published estimates for individual risk tolerances, we find that large companies with reasonably diversified shareholders should have risk tolerances that are much larger than those typically suggested in the decision analysis literature. We also find that, in contrast with what is commonly assumed in the finance literature, market prices for gambles generally do not reflect the interests of shareholders, and market-value maximization can lead to the selection of dominated alternatives.

1. Introduction

There is considerable debate about the appropriate way to think about and model risk attitudes in corporate settings. At one end of the spectrum, some decision analysts suggest assessing the utility curve of the particular manager or business unit responsible for making the decision under consideration and using this utility function when analyzing this particular decision. Other decision analysts suggest working with senior corporate officers to develop a corporate utility function that describes the firm's attitude toward risk and using this utility function for all decisions made by the firm. In contrast, finance theorists and practitioners typically suggest that firms should act in the interests of their shareholders and (therefore?) should strive to maximize the market value of the firm and be risk neutral toward "unsystematic" risks.

The first of the approaches described above—where analysts model the preferences of the manager or business unit making the decision—is normative in that it takes the preferences of the decision maker as given and applies expected utility principles thereafter. The study of oil exploration prospects by Walls et al. (1995) is representative of this approach: They recommend interviewing the decision maker to assess a risk tolerance and suggest as a "rule of thumb" a risk tolerance approximately equal to one-quarter of the exploration units' annual budget. However, there is often considerable variation in a manager's utilities even in the same firm (see, e.g., Swalm 1966 or Spetzler 1968). Using different utilities for different managers or business units may lead to inconsistencies at the corporate level. For example, one manager or business unit may reject an investment that others would happily accept.

The second approach described above—where analysts work with senior corporate officers to develop a corporate utility function for use throughout the firm—is frequently used and advocated by decision analysis practitioners (see, e.g., Spetzler 1968, Howard 1988, Bickel et al. 2002) and is sometimes referred to as the "subscription model." Here the idea is that the firm would declare its utility curve, perhaps in its corporate charter, and employees and shareholders in choosing to be associated with the firm would agree to adopt and accept, respectively, the stated preferences. Howard (1988) reports that in interviews with top executives, he typically finds risk tolerances approximately equal to one-sixth of the book value

of the firm's equity; McNamee and Celona (1990), who worked with Howard, recommend as a rule of thumb a risk tolerance equal to approximately 20% of the market value of the firm, noting that this ratio translates better between companies in different industries. Bickel et al. (2002) find similar numbers at this same level of the firm. In an empirical study estimating firm risk tolerances based on investment decisions in the oil and gas industry, Walls and Dyer (1996) find much smaller firm risk tolerance and suggest a nonlinear relationship between risk tolerance and firm size.

This subscription approach takes the legal notion of a corporation being a "fictitious individual" literally by endowing it with beliefs and preferences typically associated with individuals in our decision-making theories. In contrast, finance theorists and practitioners argue that rather than making decisions according to its own utility function, the firm should make decisions in accordance with the preferences of its shareholders. The courts have generally supported this idea, noting that management's fiduciary duty to shareholders entails a "duty of loyalty" to make decisions for the benefit of shareholders. While employees, debt holders, and other stakeholders are protected by contracts and applicable law, shareholders, as the holders of the residual claims on the firm, "receive few explicit promises. Instead they get the right to vote and the protection of fiduciary principles: the duty of loyalty and the duty of care." (Easterbrook and Fischel 1991, p. 91). The "duty of care" requires managers to act for shareholders as a prudent person would in the management of their own affairs. The "duty of loyalty" requires managers to make decisions in the interests of shareholders rather than their own interest or in the interest of other constituencies. If we take this duty of loyalty seriously, we must face some thorny issues in making this concept of shareholder-based preferences operational. While there may be cases where all shareholders unanimously endorse a particular action, there may also be conflicts between their preferences.

The goal of this paper is to study how acting in the interests of shareholders should constrain a firm's risk preferences. While the analysis of the paper was motivated by considering large, publicly held corporations, the results are also applicable to small or closely held companies or other situations where a fiduciary—for example, the trustee of an estate—makes decisions on behalf of others. To focus our discussion on risk attitudes and simplify the analysis, we will consider a single firm facing "unsystematic risks" that are independent of all other risks in the economy. The firm's goal is to make decisions in the interests of its shareholders. This framework is described in detail in §2. In §3, we study the firm's risk preferences and derive bounds on corporate risk tolerances and certainty equivalents based on the preferences of the shareholders. In §4, we consider the implications of these results for large companies. In §5, we consider the impact of efficient shareholding obtained by trading in a competitive market. Section 6 provides some concluding remarks, comparing and contrasting these results with other studies of risk attitudes in the decision analysis and finance literature.

There are many other perspectives one could take in studying corporate risk attitudes that will not be considered in detail here. One approach is to study how various contractual and environmental factors affect a firm's willingness to take risks. For example, Smith and Smithson (1990) note that bankruptcy costs, convexities in the tax code, and considerations of the firm's reputation with customers and other factors may lead a risk-neutral firm to hedge unsystematic risks. Similarly, Froot et al. (1993) show that the possibility of having to obtain expensive outside financing to fund future investments may provide an incentive for a risk-neutral firm to hedge.

Another approach to the study of corporate risk attitudes comes through agency theory. In this literature, one studies the impact of the need to provide incentives for self-interested managers. For example, Shavell (1979) and Holmström (1979) show that optimal compensation schemes lead to risk-averse managers (agents) bearing some risks, even though this arrangement may cause the managers to make decisions that are too risk averse from the perspective of the principal, here the company or its shareholders. In this context, the suboptimal allocation of risks is justified by the gains provided by improved performance incentives. While these additional perspectives raise numerous important issues, they also beg the prior question of what the firm's risk attitude should be without these effects: What should the principal's

risk preferences be? Should the firm be risk neutral if we explicitly model the costs of financial distress?

The study of risk sharing and group decision making has a long tradition in economics. Wilson's (1968) "Theory of Syndicates" studies decision making by groups in the case where members of the group write state-contingent contracts and optimally share risks. Drèze (1974) and Grossman and Hart (1979) develop sophisticated equilibrium models of firm decision making where stockholders hold proportional interests in the firm and shareholders may trade other securities; these models are nicely summarized in Magill and Quinzi (1996). Here we consider a restricted set of contracts—corresponding to stocks in a single company—and focus on the case where the firm takes the shareholdings as given and the shares may be allocated inefficiently.

2. Basic Framework

Shareholders
Suppose that a firm has n shareholders and each shareholder seeks to maximize his own expected utility. We let $u_i(x)$ denote shareholder i's utility when he receives x from the firm. If shareholder i faces uncertainty beyond his interest in the firm so that his total wealth is given by $\tilde{x} + \tilde{e}$, we will assume that the firm's gamble $\tilde{x}$ is independent of the additional uncertainty $\tilde{e}$ so that the utility function u_i can be interpreted as a derived utility $u_i(x) \equiv E[\hat{u}_i(x + \tilde{e})]$ given by taking expectations of his "true utility function" $\hat{u}_i$ over these other uncertainties ($\tilde{e}$). Because the firm gambles ($\tilde{x}$) are assumed to be independent of the shareholders' other risks ($\tilde{e}$), the firm's gambles can be thought of as "unsystematic" risks. We will assume that the shareholders and the management all agree on the probabilities associated with the firm's gambles. These assumptions are made to simplify the analysis and focus the discussion on risk attitudes.[1]

---

[1] To capture dependencies between the firm's gambles and other uncertainties affecting the shareholders and/or differences in beliefs, we would have to construct a more complex model that would obscure the discussion of risk attitudes in this paper. Here we use the term "risk attitude" as it is typically used in the decision analysis literature to refer to the preference information provided by a decision maker's utility function. Clearly, in a market setting with nonhomogenous beliefs, there are many dimensions of

We assume that the shareholders prefer more money to less and are risk averse: This implies that the utility functions u_i are increasing and concave ($u_i'(x) > 0$ and $u_i''(x) < 0$ for all x). We let $\rho_i(x) = -u_i'(x)/u_i''(x) > 0$ denote shareholder i's risk tolerance at x; the risk tolerance is the reciprocal of the standard Arrow-Pratt measure of risk aversion. Those not accustomed to thinking in terms of risk tolerances may find it helpful to consider Howard's approximate interpretation (see, e.g., Howard 1988): The risk tolerance $\rho_i(x)$ is approximately the amount r that makes shareholder i just indifferent to taking a 50-50 gamble paying $+r$ or $-r/2$ (resulting in wealth $x + r$ or $x - r/2$) or passing on the gamble (yielding wealth x for sure). This approximation is quite accurate for the exponential utility function $u(x) = -\exp(-x/\rho)$ (which has a constant risk tolerance of ρ) and is reasonably close for many commonly used utilities.

Shareholder i's certainty equivalent for a gamble $\tilde{x}$ is denoted by $CE_i[\tilde{x}]$ and is defined, in the usual way, as the $\hat{x}$ such that $u_i(\hat{x}) = E[u_i(\tilde{x})]$. Following Pratt (1964), given certain regularity conditions, we can approximate the certainty equivalent for a gamble $\tilde{x}$ with mean μ_x and σ_x^2 as

$$CE_i[\tilde{x}] = \mu_x - \frac{\sigma_x^2}{2\rho_i(\mu_x)} + o(\sigma_x^2), \qquad (1)$$

where $o(\cdot)$ denotes an error term that is zero at zero and of a smaller order than its argument. Thus, for small gambles (i.e., as $\sigma_x^2 \to 0$), the risk premium ($E[\tilde{x}] - CE_i[\tilde{x}]$) is approximately $\sigma_x^2/(2\rho_i(\mu_x))$.

Firms
The firm's problem is to choose among gambles $\tilde{x}$ on behalf of its shareholders. Let $s_i > 0$ denote investor i's share of the firm so that investor i receives $s_i\tilde{x}$ when the firm undertakes gamble $\tilde{x}$; the total shares must sum to one. We will generally assume that these shares are exogenously specified and the firm takes them as given, although in §5 we will consider a simple model where shares are traded in a competitive market. The gambles $\tilde{x}$ are amounts paid to shareholders and should be interpreted as representing the

---

"risk attitudes" (more broadly defined) that are not addressed here. As argued in §5, however, the issues discussed in this paper are relevant to and, in a sense, would be embedded in more complex models that reflect these additional features.

payoff of the firm's entire portfolio of investments, after paying employees, corporate taxes, debt holders, etc. Although management has an obligation to honor contracts with debt holders, employees, and suppliers (and the cost of doing so should be included in $\tilde{x}$), as discussed in the introduction (§1), management does not have a fiduciary duty to make decisions in the interests of these other stakeholders. Therefore, their interests are not considered as part of the firm's objectives in the analysis of this paper.

To be precise about what we mean by a firm "making decisions for the benefit of shareholders," we assume the following:

ASSUMPTION 1. COMPLETENESS. *The firm has preferences for gambles $\tilde{x}$ that can be represented by a real-valued choice function $V(\cdot)$, such that $\tilde{x}^A$ is strictly preferred to $\tilde{x}^B$ if and only if $V(\tilde{x}^A) > V(\tilde{x}^B)$.*

ASSUMPTION 2. BEST INTERESTS OF SHAREHOLDERS.
(a) *If any shareholder prefers his share of $\tilde{x}^A$ to $\tilde{x}^B$, and no shareholder prefers his share of $\tilde{x}^B$ to $\tilde{x}^A$, then the firm also prefers $\tilde{x}^A$ to $\tilde{x}^B$.*
(b) *If every shareholder is indifferent between two alternatives, the firm should also be indifferent between them.*

The completeness assumption requires a firm to be able to rank gambles. It rules out, for example, schemes that require unanimous agreement among shareholders to accept a gamble; such a scheme would lead to a partial ordering of gambles rather than a complete ordering. In essence, this assumption can be interpreted as requiring the firm to be able to make decisions. The "best interests" assumption, as a Pareto efficiency condition, provides a minimal necessary condition for these decisions to be in the best interests of shareholders by ruling out choices opposed by every shareholder.

Though the choice function $V(\tilde{x})$ ranks risky gambles, the assumptions made do not imply that the firm's choices satisfy the assumptions of expected utility theory. If, in addition, we assume that the firm wishes to follow expected utility theory, the results of Harsanyi (1955) imply that the firm's utility function must be a weighted sum of shareholder utilities

$$u(x) = \sum_{i=1}^{n} \lambda_i u_i(s_i x)$$

for some set of positive weights $\lambda_1, \lambda_2, \ldots, \lambda_n$. The firm's choice function can then be written as

$$V(\tilde{x}) = \mathrm{E}[u(\tilde{x})] = \sum_{i=1}^{n} \lambda_i \mathrm{E}[u_i(s_i \tilde{x})]. \quad (2)$$

We will not assume that the firm's preferences necessarily satisfy expected utility theory, although, as indicated in §1, this assumption is standard in decision analysis practice.[2] This assumption is controversial in group decision-making contexts; many have argued that the utility function (2) fails to capture equity issues (see, e.g., Diamond 1967, Keeney 1992). Assuming a utility function may also a priori rule out some plausible decision-making procedures, notably those based on market-value maximization and discussed in §5 below. Readers who are unfamiliar with or uncomfortable with nonexpected utility models can safely suppose that the firm has a utility function and that the choice function V is of the form of Equation (2). With the exception of the next paragraph, there is very little discussion of nonexpected utility concepts and there are few complications associated with allowing this generality. Assuming that the firm has a utility function would not strengthen any of the results that follow.

The final assumption is technical in nature and allows us to characterize the firm's risk preferences using techniques analogous to those of standard utility theory. Following Machina (1982), we assume the following.

ASSUMPTION 3. SMOOTHNESS. *The firm's choice function $V(\cdot)$ is continuous and "smooth" in the sense of being (once) Fréchet differentiable.*

Intuitively, the assumptions of continuity and differentiability require the firm's preferences to change gradually, meaning gambles that are close to one another should yield values that are close to one another. Machina (1982) shows that this assumption implies that for every gamble $\tilde{x}$, there exists a "local utility function" $u(x; \tilde{x})$ such that for any other gamble $\tilde{x}^*$,

$$V(\tilde{x}^*) - V(\tilde{x}) = \mathrm{E}[u(\tilde{x}^*; \tilde{x})] - \mathrm{E}[u(\tilde{x}; \tilde{x})] + o(\|\tilde{x}^* - \tilde{x}\|), \quad (3)$$

---

[2] One could, for example, adopt a choice function of the form $V(\tilde{x}) = \sum_{i=1}^{n} \lambda_i \mathrm{CE}_i[s_i \tilde{x}]$ for some set of positive weights $\lambda_1, \lambda_2, \ldots, \lambda_n$. This form would satisfy Assumptions 1–3, but would not be consistent with expected utility theory.

where $\|\tilde{x}^* - \tilde{x}\| = \int |dF^*(x) - dF(x)|\, dx$ (with F^* and F denoting distribution functions for $\tilde{x}^*$ and $\tilde{x}$, respectively) and $o(\text{-})$ is a function of higher order than its argument.[3] Thus, the firm evaluating small changes from a gamble $\tilde{x}$ will behave as if it has utility function $u(\text{-}; \tilde{x})$. The approximation in (3) can be interpreted as a first-order Taylor series expansion of V, expanding in the probabilities about the point $\tilde{x}$. If the firm has a utility function, then Assumption 3 is automatically satisfied, the approximation in (3) is exact, and the same local utility function $u(\text{-}; \tilde{x})$ applies everywhere. Machina (1982) shows that many of the properties and intuitions associated with utility theory carry over to this more general setting.

3. Characterizing the Firm's Risk Preferences

Practically, it would be difficult to make the interpersonal comparisons of shareholder utilities required to assess the firm's choice function $V(\text{-})$ even if we were to assume that the firm has a utility function of the form of Equation (2). We could make additional assumptions that further restrict the form of the firm's preferences (e.g., by somehow constraining the weights in Equation (2)), but such assumptions are likely to be controversial, and we can draw some strong conclusions about firm risk preferences without making any additional assumptions.

Let us define the firm's certainty equivalent for a gamble $\tilde{x}$, denoted by $\text{CE}[\tilde{x}]$, in terms of the choice function V as the $\hat{x}$ such that $V(\hat{x}) = V(\tilde{x})$. This is the standard definition of the certainty equivalent: $\hat{x}$ is the constant lump sum such that the firm is indifferent between receiving $\hat{x}$ and the gamble $\tilde{x}$. Because $V(\hat{x})$ is increasing and continuous in $\hat{x}$ for constant $\hat{x}$ (this follows from Assumptions 2 and 3), the certainty equivalent of any gamble is uniquely defined. As in standard utility theory, the certainty equivalent provides a meaningful interpretation of the values given by the choice function.

Assuming the firm's local utility function is sufficiently differentiable, we can define the firm's risk tolerance $\rho(x; \tilde{x})$ in terms of its local utility function

as $-u'(x; \tilde{x})/u''(x; \tilde{x})$, where u' and u'' denote the first and second derivatives with respect to x of the local utility function $u(x; \tilde{x})$. This definition is analogous to the standard definition of risk tolerance for utility functions and can be interpreted analogously. In particular, Machina (1982) establishes the following properties relating certainty equivalents and risk tolerances that are analogous to properties of standard utility functions derived in Pratt (1964).

1. Given certain regularity conditions, we can approximate the certainty equivalent of a gamble $\tilde{x}$ with mean μ_x and σ_x^2 as

$$\text{CE}[\tilde{x}] = \mu_x - \frac{\sigma_x^2}{2\rho(\mu_x; \mu_x)} + o(\sigma_x^2), \qquad (4)$$

where $\rho(\text{-}; \mu_x)$ denotes the firm's local risk tolerance given a sure μ_x.

2. If $\rho_1(x; \tilde{x}) \leq \rho_2(x; \tilde{x})$ for all x and all $\tilde{x}$, then $\text{CE}_1[\tilde{x}] \leq \text{CE}_2[\tilde{x}]$ for all $\tilde{x}$.

The first result is analogous to the small risk approximation given in Equation (1) and can be used similarly. The second result is analogous to part of Pratt's Theorem 1 and can be used to calculate bounds on certainty equivalents using bounds on the firm's risk tolerances. Thus, we can use an upper or lower bound on the firm's risk tolerance to calculate an upper or lower bound on the firm's certainty equivalent.

Now let us consider how shareholder i would like the firm to value gambles. Given that shareholder i receives share s_i of gamble $\tilde{x}$, he would like the firm to adopt the utility function $u_i(s_i x)$ and would want the firm to assign a certainty equivalent of $\hat{x} = \text{CE}_i[s_i \tilde{x}]/s_i$ to gamble $\tilde{x}$, because $u_i(s_i \hat{x}) = \text{E}_i[u_i(s_i \tilde{x})]$. We call this shareholder i's *target certainty equivalent*. We can similarly characterize the shareholder's *target risk tolerance* by differentiating $u_i(s_i x)$: Shareholder i would like the firm to have risk tolerance $-s_i u_i'(s_i x)/(s_i^2 u_i''(s_i x)) = \rho_i(s_i x)/s_i$. These target certainty equivalents and target risk tolerances are the certainty equivalents and risk tolerances that the firm would assign if it cared only about the preferences of this shareholder or if all shareholders had preferences identical to those of this shareholder.

In general, shareholders may disagree about the desired certainty equivalents and risk tolerances, but

---

[3] Continuity and differentiability are defined using the metric space defined by the metric $\|\tilde{x}^* - \tilde{x}\|$. As discussed in Machina (1982), this is a true metric only if the gambles have bounded support.

we can place bounds on the firm's risk tolerances and certainty equivalents as follows.

PROPOSITION 1.

(a) *For any $\tilde{x}$, the firm's risk tolerance, $\rho(x;\tilde{x})$, lies between the following bounds:*

$$\min\left\{\frac{\rho_1(s_1 x)}{s_1}, \frac{\rho_2(s_2 x)}{s_2}, \ldots, \frac{\rho_n(s_n x)}{s_n}\right\}$$

$$\leq \rho(x;\tilde{x}) \leq \max\left\{\frac{\rho_1(s_1 x)}{s_1}, \frac{\rho_2(s_2 x)}{s_2}, \ldots, \frac{\rho_n(s_n x)}{s_n}\right\}.$$

(b) *For any gamble $\tilde{x}$, the firm's certainty equivalent, $CE[\tilde{x}]$, lies between the following bounds:*

$$\min\left\{\frac{CE_1[s_1\tilde{x}]}{s_1}, \frac{CE_2[s_2\tilde{x}]}{s_2}, \ldots, \frac{CE_n[s_n\tilde{x}]}{s_n}\right\}$$

$$\leq CE[\tilde{x}] \leq \max\left\{\frac{CE_1[s_1\tilde{x}]}{s_1}, \frac{CE_2[s_2\tilde{x}]}{s_2}, \ldots, \frac{CE_n[s_n\tilde{x}]}{s_n}\right\}.$$

These bounds cannot be tightened without placing further restrictions on the choice function and the inequalities in (a) and (b) will either both hold with equality or will both be strict.

A formal proof of this proposition is given in the appendix. The bounds of Part (b) follow directly from the "best interests" Assumption 2: If all shareholders value the gamble less than some amount $\hat{x}$, then the firm must also value the gamble less than $\hat{x}$. The smoothness of V is not required for the bounds on certainty equivalents in Part (b), but is required to link risk tolerances and certainty equivalents in Part (a). Also note that these bounds would be no tighter if we assumed the firm follows expected utility theory and has a utility function of the form of (2): The upper and lower bounds on risk tolerances and certainty equivalents would be obtained by a utility function that puts zero weight on all but one shareholder.

To illustrate the structure of these bounds, let us consider its application to a hypothetical firm with three shareholders, each having a one-third interest in the firm (i.e., $s_i = 1/3$ for all i). Suppose the first shareholder has an exponential utility of the form $u_1(x) = -\exp(-(15+x)/2)$, which, if x is measured in millions of dollars, corresponds to a base wealth of $15 million and a constant risk tolerance of $2 million. The second shareholder has a power utility function of the form $u_2(x) = -(20+x)^{-5}$ corresponding

to a base wealth of $20 million and risk tolerance equal to one-sixth of his wealth $(20+x)$. The third shareholder has a power utility function of the form $u_3(x) = -(9+x)^{-2}$ corresponding to a base wealth of $9 million and risk tolerance equal to one-third of his wealth $(9+x)$.

Figure 1a shows the target risk tolerances for these three shareholders for different levels of firm income (x). According to Proposition 1, the upper and lower bounds on the firm's risk tolerance are given by the smallest envelope containing all of the shareholders' target risk tolerances. In the figure, we see that different shareholders play different roles in these bounds. For large negative amounts, the third shareholder's utility gives the lower bound and (in a range beyond that shown in the graph) the first shareholder's utility gives the upper bound. For large positive amounts, the two roles are reversed. For intermediate amounts, the second shareholder's utility generates the upper bound. Given the structure of the bounds, it is easy to see that if the shareholders' risk tolerances are all increasing (or nondecreasing) in wealth, then both the upper and lower bounding envelopes will be increasing (or nondecreasing) in x.[4]

Figure 1b illustrates the certainty equivalent bounds of the proposition in this example by showing bounds for a series of gambles that yield either nothing or some amount x with equal probabilities. Here we see that for gambles with small stakes, the expected values provide a very good approximation of certainty equivalents. From the small risk approximation of Equation (4), we see that for gambles with standard deviations that are approximately 10% of the risk tolerance, the risk premiums will be less than approximately 0.5% of this risk tolerance. Smaller gambles will have smaller risk premiums.

---

[4] Note that although every corporate risk tolerance will lie between the bounds of the proposition, not every risk tolerance function that lies between these bounds will be consistent with shareholder's preferences. For example, if the shareholder with the minimal target risk tolerances changes for different payoffs (x), the minimal risk tolerance function that is equal to the lower bound value at every point x would be more risk averse than any shareholder would desire. One can generate a lower bound on the firm certainty equivalent by using a utility function whose risk tolerance matches this pointwise minimal risk tolerance. This bound will be less than or equal to (and may be strictly less than) the lower bound on the certainty equivalent given by Part (b) of the proposition.

Figure 1a   Target Risk Tolerances for the Example with Equal Shareholdings
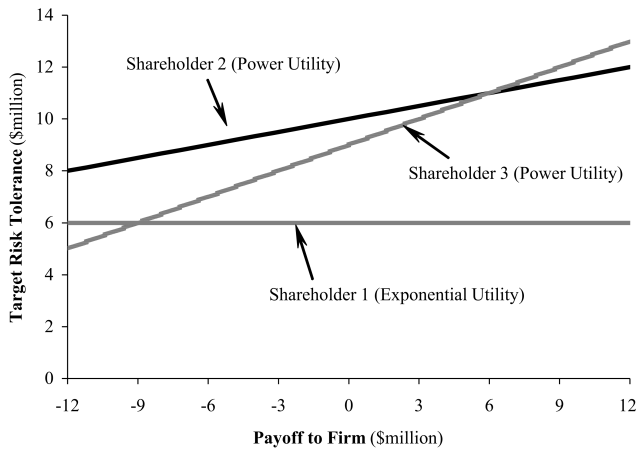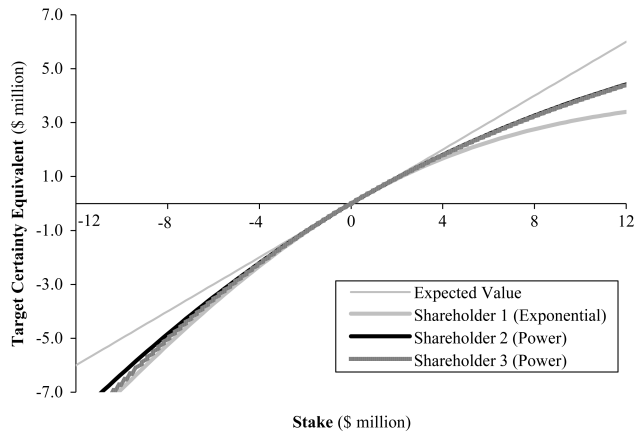

Figure 1b   Target Certainty Equivalents for the Example with Equal Shareholdings

For gambles with larger stakes, the risk premiums are larger, the expected value approximation becomes less accurate, and there is more disagreement about the firm's certainty equivalent. For example, for a gamble that is equally likely to pay $5 million or $0, we find target certainty equivalents of $1.994, $2.202, and $2.184 for the three shareholders, compared to an expected value of $2.50 million.

4.   Target Risk Tolerances for Large Corporations

To determine precise bounds on firm risk tolerances or certainty equivalents, a company would have to survey its shareholders to determine their risk preferences. For a small company (or an estate), such a survey would be feasible, and in some cases it may be necessary to get a sense for the appropriate group risk preferences. For a larger company, such a survey or poll may be unnecessary as some simple and rough calculations may be sufficient to provide adequate guidance.

To illustrate, consider a shareholder who has a current risk tolerance of $100,000 and has $10,000 invested in the stock of a large company. If the company's stock were worth, say, $10 billion dollars in total, then this shareholder would own a 0.0001% (=$10,000/$10 billion) share of the company. The target risk tolerance corresponding to this shareholder would then be $100,000/0.0001% = $100,000,000,000. Thus, this shareholder—who has a fairly sizable stake in the company and is not terribly well diversified—would want the company to behave as if it had a $100 billion risk tolerance! Here we see that, for large companies, the small fractions owned by individual investors greatly amplify their individual risk tolerances and lead to very large risk tolerances for the firm.

To further illustrate, let p_i denote shareholder i's proportional risk tolerance, i.e., his current risk tolerance as a proportion of his wealth w_i, $p_i = \rho_i/w_i$. Our example shareholder with a $100,000 risk tolerance might have a total wealth (w_i) of $600,000 and a proportional risk tolerance equal to one-sixth. If the shareholder invests some fraction f_i of his wealth (for our example shareholder, $f_i = 10,000/600,000 = 1.67\%$) in a company whose stock is worth a total of V, then the investor owns a share of the company equal to $s_i = (f_i \times w_i)/V$. The target risk tolerance corresponding to this shareholder is then given by $\rho_i/s_i = (p_i \times w_i)/((f_i \times w_i)/V) = p_i V/f_i$. Note that this target risk tolerance is independent of the shareholder's wealth and, moreover, the ratio of the target risk tolerance to the firm value $(\rho_i/s_i)/V = p_i/f_i$ is independent of the size of the firm. Our example shareholder with $p_i = 1/6$ and $f_i = 1.67\%$ would want the firm to have a risk-tolerance-to-firm-value ratio of $p_i/f_i = (1/6)/1.67\% = 10$.

We would naturally expect to find considerable variation in the risk tolerance proportions p_i and fractions f_i across different shareholders. We can use published estimates to get a sense of plausible ranges

for risk tolerance proportions. For example, French et al. (1987) and Harvey (1989) use data on investment returns to obtain estimates of aggregate relative risk aversion coefficients (the reciprocal of this risk tolerance proportion p_i) of 7.8 and 5.27, respectively. More recently, Graham and Harvey (1996) estimate relative risk aversion coefficients from the asset allocation recommendations of investment newsletters and find relative risk aversion coefficients for the investment newsletters that range from 2.6 to 24.5 with a mean of 9.33. Barsky et al. (1997) recently estimated individual risk tolerances for a large cross-section of 51- to 61-year-olds using survey questions about hypothetical income gambles and found a mean risk tolerance proportion (p_i) of 0.24 ($\approx 1/4$) and estimated a distribution of individual risk tolerance proportions with 10th, 50th, and 90th percentiles of 0.04 ($\approx 1/25$), 0.14 ($\approx 1/7.1$), and 0.53 ($\approx 1/1.9$). The fractions f_i could vary from near zero for a well-diversified shareholder to 100% for a completely undiversified shareholder. Table 1 shows how the firm-risk-tolerance-to-value ratios vary across these ranges.

For a publicly traded corporation, the upper bound on risk tolerance is likely to be very large because there may be shareholders who have small fractions of their wealth invested in the company. It is difficult to determine a precise lower bound on firm risk tolerance. A relatively risk-averse individual who is rather poorly diversified might have a proportional risk tolerance of say, $p_i = 1/10$ and a fraction $f_i = 10\%$ invested in the firm, leading to a risk-tolerance-to-firm-value ratio of 1.0. However, one could argue that such a risk-averse person should diversify their investments further and reduce the fraction of their wealth in this firm. This suggests that risk tolerances for companies with reasonably diversified shareholders should be at least as large as the value of the firm. If we expect shareholders to be better diversified, we should have even larger recommended risk tolerances.

How can we get firm risk tolerances as low as those values reported by Howard (1988), namely one-sixth of the value of the firm? Examining Table 1, we see that this would occur only if we considered shareholders who are extremely risk averse *and* also have large fractions of their wealth invested in the firm. Managers whose savings and human capital are both fully invested in their firm may qualify as such shareholders. For example, if a manager has a risk-tolerance-to-wealth ratio (p_i) of one-sixth and has 100% of his wealth invested in the company, this would lead to a target risk tolerance equal to one-sixth of the value of the firm. However, in this case, the manager's personal interest would be in conflict with those of the other shareholders and, as part of their "duty of loyalty" to shareholders, the manager should set aside consideration of his or her own personal financial stakes in deference to the interests of the shareholders (see Bagley 1995, p. 652).

5. Efficient Shareholding and Market Values

In the finance literature, it is often assumed or asserted that firms should choose among alternative investments on the basis of their market value, the price the asset would have if it were traded in a

Table 1   **Representative Firm-Risk-Tolerance-to-Value Ratios**

	p_i							
	1/1	1/2	1/3	1/6	1/10	1/15	1/20	1/25
f_i (%)								
0.1	1,000	500	333	167	100	67	50	40
1	100	50	33	17	10	6.7	5.0	4.0
2	50	25	17	8	5.0	3.3	2.5	2.0
5	20	10	6.7	3.3	2.0	1.3	1.0	0.80
10	10	5.0	3.3	1.7	1.0	0.67	0.50	0.40
25	4.0	2.0	1.3	0.67	0.40	0.27	0.20	0.16
50	2.0	1.0	0.67	0.33	0.20	0.13	0.10	0.080
67	1.5	0.75	0.50	0.25	0.15	0.10	0.075	0.060
90	1.1	0.56	0.37	0.19	0.11	0.074	0.056	0.044
100	1.0	0.50	0.33	0.17	0.10	0.067	0.050	0.040

competitive market. This is assumed, for example, in Lintner's classic paper on the Capital Asset Pricing Model when he discusses the use of the CAPM for capital budgeting (Lintner 1965). This principle of market-valuation maximization is thought to be in the best interests of the shareholders (see, e.g., Brealey and Myers 2003). While we will not develop a full model of a stock market in this paper, in this section we will consider a simple model where shareholders trade shares of a single firm. We study the effect of efficient shareholding on corporate risk tolerances and compare and contrast market values and firm certainty equivalents in this setting.

While in the previous analysis we took the shares $(s_1, s_2, \ldots, s_n)$ to be exogenously specified, now we assume that the shareholders select their shareholdings optimally in a competitive market. More specifically, suppose that each shareholder takes the share price (p) as given and chooses s_i to maximize $E[u_i(s_i(\tilde{x} - p) + \xi_i p)]$, where ξ_i is shareholder i's initial endowment of shares in the firm. The equilibrium price p is then determined by finding the price such that the optimal share holdings s_i^* sum to one. This is referred to as a "financial markets equilibrium" in the economics literature and one can show that the allocation is "constrained Pareto optimal" in that it is not possible to make shareholders better off by trading the stock of the firm (see, e.g., Magill and Quinzi 1996, Chapter 2). We call these shareholdings "efficient" for this reason. If we take the firm's choice function $V(\tilde{x})$ to be the equilibrium price p that $\tilde{x}$ would have if traded in a market in equilibrium, $V(\tilde{x})$ provides an ordering as required by Assumption 1 and, with suitable regularity conditions, $V(\tilde{x})$ can be shown to be "smooth" as required by Assumption 3. However, this $V(\tilde{x})$ does not satisfy the assumptions of utility theory and, as we will see shortly, does not satisfy the "best interests" assumption (Assumption 2).

We can characterize the equilibrium allocation of shares quite simply in the case of small gambles. If we consider a gamble $\tilde{x}$ with mean μ_x and σ_x^2 with price p, we can use Equation (1) to write the investor's certainty equivalent for his share of the gamble $s_i\tilde{x}$ as

$CE_i[s_i(\tilde{x}_i - p) + \xi_i p]$

$$= s_i(\mu_x - p) + \xi_i p - \frac{s_i^2 \sigma_x^2}{2\rho_i(s_i(\mu_x - p) + \xi_i p)} + o(s_i^2 \sigma_x^2). \quad (5)$$

Assuming that $s_i\tilde{x}$ is small (i.e., neglecting the error term and taking $\rho_i(s_i(\mu_x - p) + \xi_i p) = \rho_i(\xi_i \mu_x)$ because $p \approx \mu_x$) and considering the first-order conditions for optimality, we find that the optimal shares s_i^* satisfy

$$\mu_x - p - \frac{s_i^* \sigma_x^2}{\rho_i(\xi_i \mu_x)} = 0 \quad (6)$$

and are thus given by

$$s_i^* = \rho_i(\xi_i \mu_x)\frac{\mu_x - p}{\sigma_x^2}. \quad (7)$$

Noting that in equilibrium the shares must sum to one, we have

$$\frac{\mu_x - p}{\sigma_x^2} = \frac{1}{\rho(\mu_x)}, \quad (8)$$

where $\rho(\mu_x) \equiv \sum_{i=1}^n \rho_i(\xi_i \mu_x)$. Substituting into (6), we find that

$$s_i^* = \frac{\rho_i(\xi_i \mu_x)}{\rho(\mu_x)}, \quad (9)$$

so that, in equilibrium, each investor would hold shares in small gambles in proportion to their base risk tolerance $\rho_i(\xi_i \mu_x)$. If we assume that the shares are held according to this optimum, the target risk tolerance for shareholder i is $\rho_i(\xi_i \mu_x)/s_i^* = \rho_i(\xi_i \mu_x)/(\rho_i(\xi_i \mu_x)/\rho(\mu_x)) = \rho(\mu_x)$. Thus, with an efficient allocation of shares, for small gambles, the target risk tolerances are identical for every investor and the bounds on risk tolerances given by Proposition 1 collapse to ρ at this base level.

For instance, in our three-shareholder example, with a zero-mean gamble the individual base risk tolerances are 2 for the investor with an exponential utility and 3.33 and 3 for the two investors with power utilities. The total base risk tolerance for a zero-mean gamble is thus 8.33 and the optimal shares s_i^* for these three shareholders would be 24% (=2/8.33), 40% = (3.33/8.33), and 36% (=3/8.33), as opposed to equal weightings assumed earlier. With these shareholdings, we obtain the bounds on risk tolerances and certainty equivalents as shown in Figures 2a and 2b. Here we see that the effect of an efficient allocation of shares is to draw the upper and lower bounds on firm risk tolerances closer together in the vicinity of the base level around which the shares are optimized. Comparing Figures 1b and 2b, we see that the bounds on certainty equivalents are also narrower.

Figure 2a   Target Risk Tolerances with "Optimal" Fixed Shares
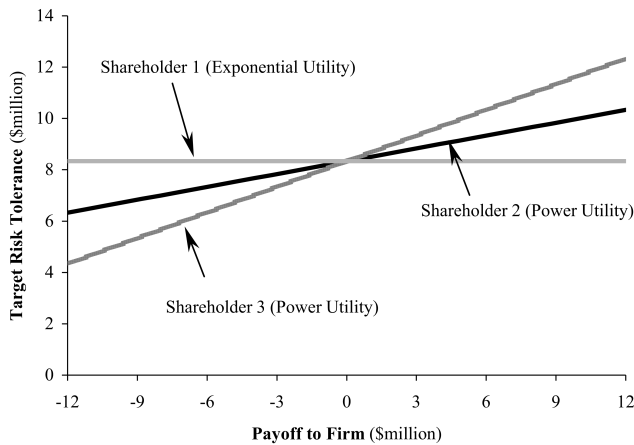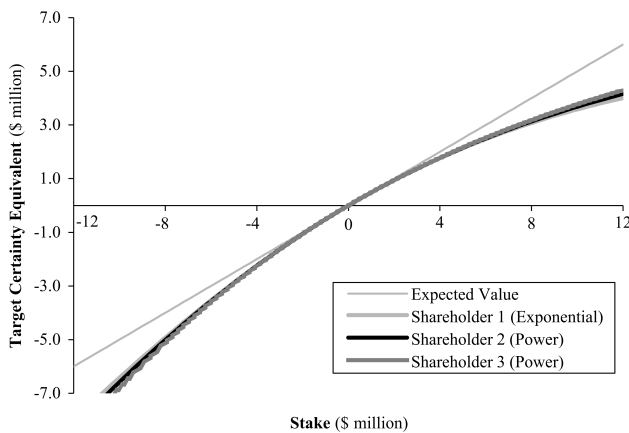

Figure 2b   Target Certainty Equivalents with "Optimal" Fixed Shares

These shares are optimal only for small risks, and agreement about target risk tolerances and certainty equivalents generally holds only for small risks.[5] If, for example, the three example shareholders trade shares of a 50-50 gamble for $5 million or zero (starting with equal endowments), in equilibrium they would hold 22.85%, 39.72%, and 37.44% and have target certainty equivalents of $2.148, $2.147, and $2.146 (respectively), indicating a slight disagreement about the value of the gamble. (These amounts were found numerically.) Note that, perhaps contrary to initial intuition, inefficient shareholdings may actually lead to higher values than efficient shareholdings.

[5] In the special case where all shareholders have exponential utilities and have constant risk tolerances, the small risk allocation of shares is optimal for all risks. See Wilson (1968).

As discussed in the previous section, with equal shareholdings this 50-50 gamble for $5 million or zero had target certainty equivalents of $1.994, $2.202, and $2.184. Thus, with equal shareholdings, a value of $2.20 million would be in the range of values consistent with acting in shareholders' best interests, but with the efficient shareholdings the upper bound drops to $2.148 million. With efficient shareholdings the shareholders with the higher risk tolerances take a larger share of the gamble and consequently assign a higher risk premium.

It is interesting to compare the market prices for gambles in this model with firm risk tolerances. Rearranging Equation (8), we see that the "risk premium" in the market price $(\mu_x - p)$ is approximately $\sigma_x^2 / \rho(\mu_x)$. Comparing this with the small risk approximation of certainty equivalents in Equation (4), we see that the risk premium in the firm's certainty equivalent is approximately $\sigma_x^2 / 2\rho(\mu_x)$. Thus, market prices for small gambles reflect a risk tolerance that is half of that for a firm that shares risk efficiently. For a larger gamble that is equally likely to pay $0 or $5 million, the market-clearing price for the gamble is $1.80 million (a risk premium of $0.70), while the bounds on the certainty equivalent range from $2.146 to $2.148 million (a risk premium of approximately $0.35). If we were to rank gambles by market prices, we would prefer a certain $2 million to a 50-50 chance at $0 or $5 million, even though all shareholders would prefer the gamble. Market-price maximization is thus inconsistent with shareholder interests and can lead to the selection of Pareto-dominated alternatives: A firm choice function $V(\tilde{x})$ based on equilibrium prices does not satisfy the best interests condition (Assumption 2). Intuitively, market prices reflect the value of the next share of a gamble and do not reflect the total benefit associated with holding the gamble.[6]

Although we have not developed a full-fledged model of the securities market, this conflict between market-value maximization and shareholder interests

[6] If the firm holding the gamble were given the choice between an additional 1% stake in the gamble (increasing the $5 million prize to $5.05 million) and a 1% stake in the sure $2 million (increasing the winning stake to $5.02 million and the losing stake from 0 to $0.02 million), the shareholders would be willing to pay more for the incremental share of the sure $2 million.

persists in more complex models of securities markets. A similar conflict was noted in the CAPM in a series of papers in the early 1970s (Stiglitz 1972, Long 1972, Jensen and Long 1972). More generally, if markets are complete in that every uncertainty can be perfectly hedged by trading marketed securities, there is no difference between market values and properly calculated certainty equivalents (see, e.g., Smith and Nau 1995). If, however, markets are incomplete and firms take risks that cannot be hedged, then risk preferences must play a role, and firms should not strive to maximize market values. Smith and Nau (1995) develop a valuation procedure that recognizes opportunities to trade securities, but focuses on maximizing certainty equivalents rather than market prices. In that valuation procedure, market risks are valued using modern "risk-neutral" valuation methods based on market prices and private (nonmarket) risks are valued by calculating certainty equivalents using a corporate risk utility function. The results of this paper provide some guidance in selecting an appropriate utility function for the valuation procedure developed in Smith and Nau (1995).

6. Discussion

The bounds in §3 and the illustrative calculations of §4 suggest that if a company wants to serve the interests of reasonably diversified shareholders, then it should have a risk tolerance at least as large as the value of the firm itself. In saying this, we take "reasonably diversified shareholders" to mean shareholders having no more than 5% of their wealth invested in the stock of the firm. If we take "reasonably diversified" to imply greater diversification and lower fractions invested in the firm, we obtain larger risk tolerances. As discussed in §5, if the shares are efficiently allocated, then we have narrower bounds on allowed corporate risk tolerances and larger minimal values. These recommended values—even the minimal values—are much higher than those typically recommended in the decision analysis literature. In this section, we review some possible reasons for these discrepancies and discuss the implications of our analysis for practice.

First, the risk tolerances reported here could be too high because the simple model considered in this paper fails to capture some important

real-world complications that induce risk aversion. Specifically, we have assumed that the costs of financial distress—including the deadweight financial costs of bankruptcy, the cost of obtaining expensive outside financing to fund future investments, costs due to damage to the firm's reputation, loss of employees, etc.—are included in the payoffs of the gambles. These costs may not have been explicitly included in the gambles used to assess risk attitudes in these other studies, and the reported risk tolerances may implicitly reflect these unmodeled costs. However, Bickel's (1999) analysis suggests that these costs are not likely to be sufficiently large to explain the discrepancies between the numbers suggested here and those reported in the decision analysis literature.

A second explanation of this discrepancy in risk tolerances is that the lower values reported in the decision analysis literature reflect the interests of the firm's managers rather than its shareholders. As shown in Table 1, one can obtain risk tolerances close to those reported by Howard (1988) by considering the interests of a manager who has 100% of his wealth invested in the company. These are the kinds of results that might be predicted by agency theory. In this literature, managers are assumed to make decisions for their personal benefit rather than that of the firm or its shareholders, and a key challenge is the design of performance contracts that align the interests of the principal (the firm) and the agent (the manager). However, as argued in §4, if a manager's financial interests are in conflict with those of the other shareholders, as part of their "duty of loyalty" to shareholders, the manager should set aside consideration of his or her own personal financial stakes when making corporate decisions.

Alternatively, one might (and I believe should) interpret the difference between the corporate risk tolerances reported in the decision analysis literature and the higher ones suggested here as the difference between descriptive and normative levels of risk tolerance. Here the "normative" power derives the simple decision-theoretic arguments based on the legal principle that firms should make decisions in the best interests of the shareholder. As discussed in Rabin and Thaler (2001), individuals often exhibit a level of risk aversion for small isolated gambles that implies "absurdly severe risk aversion" in larger

or aggregated gambles. This excessive risk aversion for small isolated gambles, Rabin and Thaler argue, reflects two fundamental psychological phenomena— loss aversion and mental accounting. Loss aversion refers to the tendency for the pain of a loss, relative to some reference point, to be felt more acutely than a corresponding gain. Mental accounting (or narrow framing) refers to the tendency to view gambles in isolation rather than in a broader context or as one of many gambles that one takes over time, and compounds the effects of loss aversion. My belief is that most managers try to do what they think is in the best interest of the firm and its shareholders, but fall prey to these psychological traps with the help of performance and evaluation systems that reinforce a narrow, loss-averse attitude. (See Kahneman and Lovallo 1993 for further discussion of this point.)

Experienced decision analysts are well aware of these psychological phenomena and, when eliciting utilities, they will often push their clients to adopt a broader frame of reference rather than accept initial overly risk-averse assessments. For example, Spetzler (1968) describes a management team who in the course of developing a corporate risk policy had a long discussion about the availability of capital and the possibility of "playing the odds." This discussion led the managers to be comfortable with less risk-averse policies. Similarly, David Bell and Howard Raiffa instruct their students to conduct their analyses in terms of final asset positions rather than gains or losses from a reference point (see Bell et al. 1988). This reframing of the problem helps avoid these psychological traps and reduces risk aversion. If we adopt the legal principle that firms should make decisions in the interests of their shareholders, then we should push managers to consider and adopt the perspective of shareholders and to think through the effects of diversification when considering risk tradeoffs. The goal of this paper has been to describe the implications of this perspective on the firm's risk preferences.

While the results of this paper do not suggest a specific risk tolerance for any firm, the minimum levels suggested by the lower bounds in §3 and §4 are large enough to suggest that most decision analyses should not be very sensitive to risk attitudes in the specified ranges. In the small risk approximation of certainty equivalents of Equation (4), we see that the risk premium for a gamble with variance σ_x^2 is approximately $\sigma_x^2/2\rho$, where ρ is the firm's risk tolerance. If we take the value of the firm (V) as a lower bound on ρ, we can take $\sigma_x^2/2V$ as a rough upper bound on risk premiums. In applications, one can check whether risk premiums in this range would have a significant impact on the results of the analysis. If not, then it may be sufficient to simply work with expected values. Howard (1988, p. 689) reports that he finds the ability to capture risk preference "a matter of real practical concern in only 5%–10% of business decision analyses" and this is based on his view that corporate risk tolerances are typically about 16%–20% of the value of the firm. With the larger risk tolerances suggested here, risk aversion should matter less frequently. However, with large-stakes decisions, risk aversion may significantly impact the results and must be considered carefully. Such large-stakes decisions may entail the use of a corporate utility function and, if the stakes are truly significant, the decision-making process may also require discussions with shareholders and perhaps a vote to be sure that shareholder preferences are well understood and represented.

The idea that large companies should be risk neutral towards unsystematic risks is, of course, familiar in the finance literature. As in the finance literature, we start from the premise that firms should make decisions in the best interests of shareholders. However, we follow a different path to the conclusion of (near) risk neutrality. Specifically, we do not assume that investors hold shares optimally in some equilibrium model and we do not assume that firms should choose among investments according to their market value. In fact, we show that the market value criterion is inconsistent with making decisions in the best interests of the shareholders. Here we assume only that firms should make decisions in the interests of shareholders. For a large firm with reasonably diversified shareholders, this alone is enough to imply that the firm should be essentially risk neutral towards all but the largest of unsystematic risks.

Acknowledgments

Appendix

PROOF OF PROPOSITION 1. (b) We focus on the upper bound; a similar argument holds for the lower bound. By the definition of the certainty equivalent, for any certain amount $\hat{x} > \text{CE}_i[s_i\tilde{x}]/s_i$, shareholder i would rather receive his share $s_i\hat{x}$ of $\hat{x}$ rather than his share $s_i\tilde{x}$ of the gamble $\tilde{x}$. Thus, all shareholders would unanimously prefer receiving their share of any amount $\hat{x} > \overline{\text{CE}}[\tilde{x}] = \{\text{CE}_1[s_1\tilde{x}]/s_1, \text{CE}_2[s_2\tilde{x}]/s_2, \ldots, \text{CE}_n[s_n\tilde{x}]/s_n\}$ to receiving their share of the gamble $\tilde{x}$. By Assumption 2, $\overline{\text{CE}}[\tilde{x}]$ is an upper bound on the firm's certainty equivalent $\text{CE}[\tilde{x}]$. If the constant sum $\hat{x}$ is less than $\overline{\text{CE}}[\tilde{x}]$, then by considering a utility function of the form of (2) and putting enough utility weight λ_i on a shareholder a target certainty equivalent greater than $\hat{x}$, you can have the firm prefer $\tilde{x}$ to $\hat{x}$. Thus, the bound of the proposition cannot be tightened without further constraining the choice function.

If the shareholders do not all have the same target certainty equivalent, then the firm's certainty equivalent cannot be equal to $\overline{\text{CE}}[\tilde{x}]$, because at least one shareholder (any shareholder not achieving this upper bound) would prefer receiving their share of $\overline{\text{CE}}[\tilde{x}]$ for certain and no other shareholder would have the reverse preference violating Assumption 2. Thus, the firm's certainty equivalent will be less than $\overline{\text{CE}}[\tilde{x}]$ except in the case where the target certainty equivalents are all equal and the upper and lower bounds collapse to a single value.

(a) This follows from Part (b): If the firm's risk tolerance exceeds the bounds of Part (a) of the proposition, then we can construct a gamble whose certainty equivalent exceeds the bounds of Part (b) of the proposition. To formalize this construction, suppose $\rho(x^*; \tilde{x}^*) > \max\{\rho_1(s_1x^*)/s_1, \ldots, \rho_n(s_nx^*)/s_n\}$ at some point x^* and for some gamble $\tilde{x}^*$. Let i^* denote a shareholder achieving the maximum target risk tolerance, and let X be a nondegenerate interval containing x^* where i^* is maximal. (There must be some i^* that is maximal at x and is maximal over some nondegenerate interval containing x.) For all gambles $\tilde{x}$ with outcomes restricted to X, shareholder i^*'s target certainty equivalent $(\text{CE}_{i^*}[s_{i^*}\tilde{x}]/s_{i^*})$ is greater than or equal to that of any other shareholder. (This follows from Pratt 1964, Theorem 1.)

Let $u(-; \tilde{x}^*)$ denote the firm's local utility function at $\tilde{x}^*$ and let $u_{i^*}(-)$ denote shareholder i^*'s utility function. Because $\rho(x^*; \tilde{x}^*) > \rho_{i^*}(s_{i^*}x^*)/s_{i^*}$, $u(-; \tilde{x}^*)$ is not "more concave" than $u_{i^*}(-)$ on X. (This also follows from Pratt 1964, Theorem 1.) This implies (see, e.g., Royden 1968, p. 108) that there exist $x_1 < x_2 < x_3$ in X and q such that

$$\frac{u(x_2; \tilde{x}^*) - u(x_1; \tilde{x}^*)}{u(x_3; \tilde{x}^*) - u(x_1; \tilde{x}^*)} < q < \frac{u_{i^*}(s_{i^*}x_2) - u_{i^*}(s_{i^*}x_1)}{u_{i^*}(s_{i^*}x_3) - u_{i^*}(s_{i^*}x_1)}. \quad (10)$$

The first inequality implies that the local utility function $u(-, \tilde{x}^*)$ would prefer a gamble, call it $\tilde{z}$, that yields x_3 with probability q and x_1 with probability $(1-q)$ to a sure x_2, because $u(x_2; \tilde{x}^*) < q\, u(x_3; \tilde{x}^*) + (1-q)\, u(x_1; \tilde{x}^*)$. The second inequality implies that shareholder i^* has the reverse preference (preferring his share of the sure x_2 to his share of the gamble $\tilde{z}$). Because this shareholder has the largest target certainty equivalent, all other shareholders would have the same preference.

Now we need to place the gamble $\tilde{z}$ in the vicinity of $\tilde{x}^*$, where the local utility function $u(-, \tilde{x}^*)$ applies. Let $\tilde{y}_1(p)$ be the gamble that yields $\tilde{z}$ with probability p and $\tilde{x}^*$ with probability $(1-p)$, and let $\tilde{y}_2(p)$ be the gamble that yields x_2 with probability p and $\tilde{x}^*$ with probability $(1-p)$. Following Equation (3), we have

$$\frac{d}{dp}[V(\tilde{y}_1(p)) - V(\tilde{y}_2(p))]_{p=0}$$
$$= q\, u(x_3; \tilde{x}^*) + (1-q)\, u(x_1; \tilde{x}^*) - u(x_2; \tilde{x}^*) > 0,$$

with the inequality following from (10). Because V is "smooth" in the sense of Assumption 3, for some $p^* > 0$ we have $V(\tilde{y}_1(p^*)) > V(\tilde{y}_2(p^*))$; i.e., the firm prefers $\tilde{y}_1(p^*)$ to $\tilde{y}_2(p^*)$. As noted above, however, every shareholder prefers x_2 to the gamble $\tilde{z}$ and, because the shareholders follow expected utility theory, they prefer $\tilde{y}_2(p)$ to $\tilde{y}_1(p)$ for any probability p. Thus, if $\rho(x^*, \tilde{x}^*)$ exceeds all of the shareholders' target risk tolerances at x^*, we can construct a pair of gambles $\tilde{y}_1(p^*)$ and $\tilde{y}_2(p^*)$ such that the firm prefers $\tilde{y}_1(p^*)$ to $\tilde{y}_2(p^*)$ and every shareholder has the reverse preference. This contradicts our assumption that the firm makes decisions in the best interests of shareholders (Assumption 2). $\square$

References

Bagley, C. E. 1995. *Managers and the Legal Environment: Strategies for the 21st Century*, 2nd ed. West Publishing Company, St. Paul, MN.

Barsky, R. B., F. T. Juster, M. S. Kimball, M. D. Shapiro. 1997. Preference parameters and behavioral heterogenity: An experimental approach in the health and environment study. *Quart. J. Econom.* **112** 537–579.

Bell, D. E., H. Raiffa, A. Tversky. 1988. Descriptive, normative, and prescriptive interactions in decision making. D. E. Bell, H. Raiffa, A. Tversky, eds. *Decision Making: Descriptive, Normative and Prescriptive Interactions*. Cambridge University Press, Cambridge, U.K.

Bickel, J. E. 1999. The corporate contractual system and normative corporate risk attitude. Ph.D. dissertation, Department of Engineering-Economics Systems and Operations Research, Stanford University, Stanford, CA.

Bickel, J. E., D. Fishman, C. S. Spetzler. 2002. Corporate risk tolerance: Taking the right risks. Strategic Decisions Group, Menlo Park, CA.

Brealey, R. A., S. C. Myers. 2003. *Principles of Corporate Finance*, 7th ed. McGraw-Hill, New York.

Diamond, P. A. 1967. Cardinal welfare, individualistic ethics, and interpersonal comparison of utility: Comment. *J. Political Econom.* **75** 765–766.

Drèze, J. H. 1974. Investment under private ownership: Optimality, equilibrium and stability. J. H. Drèze, ed. *Allocation Under Uncertainty: Equilibrium and Optimality*. Wiley, New York, 129–165.

Easterbrook, F. H., D. R. Fischel. 1991. *The Economic Structure of Corporate Law*. Harvard University Press, Cambridge, MA.

French, K. R., G. W. Schwert, R. F. Stambaugh. 1987. Expected stock returns and volatility. *J. Financial Econom.* **19** 3–30.

Froot, K. A., D. S. Scharfstein, J. C. Stein. 1993. Risk management: Coordinating corporate investment and financing policies. *J. Finance* **5** 1629–1658.

Graham, J. C., C. R. Harvey. 1996. Market timing ability and volatility implied in investment newsletters' asset allocation recommendations. *J. Financial Econom.* **42** 397–421.

Grossman, S., O. Hart. 1979. A theory of competitive equilibrium in stock market economies. *Econometrica* **47** 293–330.

Harsanyi, J. C. 1955. Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *J. Political Econom.* **63** 309–321.

Harvey, C. R. 1989. Time-varying conditional covariances in tests of asset pricing models. *J. Financial Econom.* **24** 289–317.

Holmström, B. 1979. Moral hazard and observability. *Bell J. Econom.* **10** 74–91.

Howard, R. A. 1988. Decision analysis: Practice and promise. *Management Sci.* **34** 679–695.

Jensen, M. C., J. B. Long, Jr. 1972. Corporate investment under uncertainty and Pareto optimality in the capital markets. *Bell J. Econom. Management Sci.* **3** 151–174.

Kahneman, D., D. Lovallo. 1993. Timid choices and bold forecasts: A cognitive perspective on risk taking. *Management Sci.* **39** 17–31.

Keeney, R. L. 1992. On the foundations of prescriptive decision analysis. W. Edwards, ed. *Utility Theories: Measurements and Applications*. Kluwer Academic Publishers, Boston, MA.

Long, J. B., Jr. 1972. Wealth, welfare, and the price of risk. *J. Finance* **27** 419–433.

Lintner, J. 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Rev. Econom. Statist.* **47** 13–37.

Machina, M. J. 1982. "Expected Utility" analysis without the independence axiom. *Econometrica* **50** 277–324.

Magill, M., M. Quinzi. 1996. *Theory of Incomplete Markets*, Vol. 1. MIT Press, Cambridge, MA.

McNamee, P., J. Celona. 1990. *Decision Analysis with Supertree*, 2nd ed. Scientific Press, San Francisco, CA.

Pratt, J. W. 1964. Risk aversion in the small and in the large. *Econometrica* **32** 122–136.

Rabin, M., R. H. Thaler. 2002. Anomalies: Risk aversion. *J. Econom. Perspectives* **15** 219–232.

Royden, H. L. 1968. *Real Analysis*, 2nd ed. MacMillan, New York.

Shavell, S. 1979. Risk sharing and incentives in the principal and agent relationship. *Bell J. Econom.* **10** 55–73.

Smith, C. W., C. W. Smithson. 1990. *Managing Financial Risks*. Harper and Row, New York.

Smith, J. E., R. F. Nau. 1995. Valuing risky projects: Option pricing theory and decision analysis. *Management Sci.* **41** 795–816.

Spetzler, C. S. 1968. The development of a corporate risk policy for capital investment decisions. *IEEE Trans. Systems Sci. Cybernetics* **4** 279–300.

Stiglitz, J. E. 1972. On the optimality of the stock market allocation of investment. *Quart. J. Econom.* **86** 25–60.

Swalm, R. O. 1966. Utility theory—Insights into risk taking. *Harvard Bus. Rev.* **44** 123–136.

Walls, M. R., J. S. Dyer. 1996. Risk propensity and firm performance: A study of the petroleum exploration industry. *Management Sci.* **42** 1004–1021.

Walls, M. R., T. Morahan, J. S. Dyer. 1995. Decision analysis of exploration opportunities in the onshore United States at Phillips Petroleum Company. *Interfaces* **25**(6) 39–56.

Wilson, R. B. 1968. The theory of syndicates. *Econometrica* **36** 119–132.