Anniversary Article

Decision Analysis in *Management Science*

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As part of the 50th anniversary of *Management Science*, the journal is publishing articles that reflect on the past, present, and future of the various subfields the journal represents. In this article, we consider decision analysis research as it has appeared in *Management Science*. After reviewing the foundations of decision analysis and the history of the journal's decision analysis department, we review a number of key developments in decision analysis research that have appeared in *Management Science* and offer some comments on the current state of the field.

Key words: decision analysis; probability assessment; utility theory; game theory

1. Introduction

Management Science (*MS*) has played a distinguished and distinctive role in the development of decision analysis. As part of *Management Science*, the decision analysis department has focused on papers that consider the use of scientific methods to improve the understanding or practice of managerial decision making. The current departmental statement reads as follows:

> The decision analysis department publishes articles that create, extend, or unify scientific knowledge pertaining to decision analysis and decision making. We seek papers that describe concepts and techniques for modeling decisions as well as behaviorally oriented papers that explain or evaluate decisions or judgments. Papers may develop new theory or methodology, address problems of implementation, present empirical studies of choice behavior or decision modeling, synthesize existing ideas, or describe innovative applications. In all cases, the papers must be based on sound decision-theoretic and/or psychological principles....
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> Decision settings may consist of any combination of certainty or uncertainty; competitive or noncompetitive situations; individuals, groups, or markets; and applications may include managerial decisions in business or government.

According to Hopp's counts (Hopp 2004*MS*),[1] *Management Science* has published 590 decision analysis papers in the period from 1954 to 2003, accounting

[1] We highlight papers that have appeared in *Management Science* by including "*MS*" after the publication year in the in-text citation.

for 12% of the papers in *Management Science* and 17% of the "most-cited" papers (those receiving 50 or more cites). However, given the interdisciplinary nature of the field, it is difficult to draw sharp boundaries and determine precisely what counts as "decision analysis."

Following Bell et al. (1988), we can distinguish among three different perspectives in the study of decision making. In the *normative* perspective, the focus is on rational choice and normative models are built on basic assumptions (or axioms) that people consider as providing logical guidance for their decisions. In the domain of decision making under risk or uncertainty, the expected utility model of von Neumann and Morgenstern (1944) and the subjective expected utility model of Savage (1954) are the dominant normative models of rational choice. In the domain of judgments and beliefs, probability theory and Bayesian statistics, in particular, provide the normative foundation.

The *descriptive* perspective focuses on how real people actually think and behave. Descriptive studies may develop mathematical models of behavior, but such models are judged by the extent to which their predictions correspond to the actual choices people make. One of the most prominent descriptive models of decision making under uncertainty is the Prospect Theory model of Kahneman and Tversky (1979), later refined in Tversky and Kahneman (1992). This model captures many of the ways in which people deviate from the normative ideal of the expected utility model in a reasonably parsimonious form.

The *prescriptive* perspective focuses on helping people make better decisions by using normative models, but with awareness of the limitations and descriptive realities of human judgment. For example, we might build a mathematical model to help a firm decide whether to undertake a particular R&D project. Such a model may not include all of the uncertainties, competitive effects, and sources of value that one might expect a fully "rational" individual or firm to consider. It would likely include approximations and short cuts that make the model easier to formulate, assess, and solve. Descriptive research on decision making would help the analysts understand, for example, which model inputs (e.g., probabilities or utilities) can be reliably assessed and how these inputs might be biased. Prescriptive models are evaluated pragmatically: Do the decision makers find them helpful? Or, what is more difficult to ascertain, do they help people make better decisions?

Decision analysis is primarily a prescriptive discipline, built on normative and descriptive foundations. In our review of decision analysis in *Management Science*, we emphasize its prescriptive role, but we also discuss normative and descriptive developments that have advanced prescriptive methodologies and applications. We begin in §2 with the early history of decision analysis into the late 1960s when the decision analysis department at *Management Science* was created. In §3, we discuss the history of the decision analysis department at *Management Science*, describing the editorial structure from 1970 to the present and reviewing the number of decision analysis articles published. In §4, we discuss some of the decision analysis research that has been published in *Management Science*, focusing on developments in probability assessment (§4.1), utility assessment (§4.2), and in game theory (§4.3). In our discussion of the developments of the field, we highlight specific developments that have appeared in *Management Science* but do not strive to discuss all of the decision analysis research that has appeared in the journal or decision analysis research published elsewhere. Our goal is to give a flavor of the decision analysis research that has appeared in *Management Science* in its first 50 years and some of the debates that have influenced the field. In §5, we conclude and look ahead.

2. The Early History of Decision Analysis[2]

The normative foundations of decision analysis can be traced back at least as far as Bernoulli (1738)

and Bayes (1763). Bernoulli was concerned with the fact that people generally do not follow the expected value model when choosing among gambles, in particular when buying insurance. He proposed the expected utility model with a logarithmic utility function to explain these deviations from the expected value model. Bayes was interested in the revision of probability based on observations and proposed an updating procedure that is now known as Bayes Theorem.

Ramsey (1931) recognized that the notions of probability and utility are intrinsically intertwined and showed that subjective probabilities and utilities can be inferred from preferences among gambles. Ramsey's essays did not have much influence when they were published but they are now much appreciated: INFORMS's Decision Analysis Society awards the Ramsey Medal to recognize and honor lifetime contributions to the field. DeFinetti (1937) followed a similar path by developing a system of assumptions about preferences among gambles that allowed him to derive subjective probabilities for events. His interest was primarily in the representations of beliefs as subjective probabilities, not in the derivation of utilities.

The publication of the *Theory of Games and Economic Behavior* by von Neumann and Morgenstern (1944) attracted a great deal of attention and was a major milestone in the history of decision analysis and economics. While the primary purpose of von Neumann and Morgenstern's book was to lay the foundation for the study of games, it also established the foundation for decision analysis in the process. Specifically, in an appendix to the second edition of the book (published in 1947) von Neumann and Morgenstern provided an axiomatization of the expected utility model, showing that a cardinal utility function could be created from preferences among gambles. Their analysis took the probabilities in the decision problem as given and their axioms led to the conclusion that decision makers should make decisions to maximize their expected utility. The decision-making framework of von Neumann and Morgenstern is now referred to as the *expected utility* (EU) model.

In *The Foundations of Statistics* (Savage 1954), Savage extended von Neumann and Morgenstern's expected utility model to consider cases in which the probabilities are not given. While Savage was greatly influenced by the work of von Neumann and Morgenstern, his background was in statistics rather than economics, and his goal was to provide a foundation for a "a theory of probability based on the personalistic view of probability derived mainly from the work of DeFinetti (1937)" (Savage 1954, p. 5). Savage proposed a set of axioms about preferences among gambles that enabled him to simultaneously derive the existence of subjective probabilities for events

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[2] More complete discussions of the early development of the (subjective) expected utility framework may be found in Arrow (1951a), Raiffa (1968), von Winterfeldt and Edwards (1986), Wakker (1989, §2), and Fishburn and Wakker (1995*MS*).

and utilities for outcomes, combining the ideas of utility theory from economics and subjective probability from statistics into what is now referred to as the *subjective expected utility* (SEU) model.

Although von Neumann and Morgenstern's and Savage's EU models have had an enormous impact on decision analysis and decision theory, not everyone found their axioms compelling. Allais (1953) presented a now famous example where his preferences and those of many others, including Savage himself, violated the axioms of utility theory. Savage (1954, pp. 100–104) acknowledged the descriptive appeal of Allais' example, but did not concede the normative point, writing:

> A person who has tentatively accepted a normative theory must consciously study situations in which the theory seems to lead him astray; he must decide for each by reflection—deduction will typically be of little relevance—whether to retain his initial impression of the situation or to accept the implications of the theory for it. (Savage 1954, p. 102)

Savage proposed an alternative way of viewing Allais's problem that forced him to reconsider his initial impressions in favor of the preferences prescribed by EU theory. Examples like these helped clarify the distinction between the normative, descriptive, and prescriptive roles of the EU model in the 1950s.

With von Neumann and Morgenstern's and Savage's utility models providing the normative foundations, Edwards's "The Theory of Decision Making" (1954) launched the descriptive study of decision making as a new research area in psychology. In this paper Edwards surveyed utility concepts developed in economics and statistics and made them accessible to psychologists. Other psychologists, including Clyde Coombs at the University of Michigan and Duncan Luce at Harvard, joined Edwards (then at Hopkins, later at Michigan) in the study of behavioral decision making, taking the SEU framework as the "gold standard" or benchmark for comparison and examining deviations from this ideal. Edwards and Coombs and their students at Michigan (including Sarah Lichtenstein, Larry Phillips, Paul Slovic, and Amos Tversky) studied biases and heuristics in judgment and decision making. For example, Edwards and Phillips studied Bayesian inference and found that people tend to revise their opinion less strongly than prescribed by Bayes Theorem (Phillips and Edwards 1966, Phillips et al. 1966). Other research on probability biases and heuristics soon followed, spearheaded by Amos Tversky and Daniel Kahneman (for a summary, see Tversky and Kahneman 1974, Kahneman et al. 1982).

The 1960s saw the emergence of decision analysis, building on the prescriptive power of the SEU model and Bayesian statistics. Howard Raiffa, Robert

Schlaifer, and John Pratt at Harvard, and Ronald Howard at Stanford emerged as leaders in these efforts. Schlaifer wrote *Probability and Statistics for Business Decisions* (Schlaifer 1959), which espoused Bayesian, decision analytic principles for business decisions. Raiffa and Schlaifer's *Applied Statistical Decision Theory* (1961) provided a detailed mathematical treatment of decision analysis focusing primarily on Bayesian statistical models. Pratt (1964) made major contributions to the theory of utility for money, formalizing a measure of risk aversion, studying specific forms of utility functions, and considering properties of certainty equivalents (the selling price for a risky investment) and the demand for risky investments as related to this risk-aversion measure. Pratt et al. (1964, 1965) and Howard (1965) provided introductory expositions on statistical decision theory aimed at statistics and engineering audiences, respectively.

Howard first used the term "decision analysis" in his paper "Decision Analysis: Applied Decision Theory" (Howard 1966). Shortly thereafter, Howard published a paper "The Foundations of Decision Analysis" (Howard 1968) that laid out a process that he called the "decision analysis cycle" for solving decision problems. Raiffa's book *Decision Analysis: Introductory Lectures* (Raiffa 1968) provided a detailed and practically oriented introductory text which discussed decision trees, the use of subjective probabilities, utility theory, and decision making by groups. About this same time, Howard and James Matheson founded the Decision Analysis Group at the Stanford Research Institute (later SRI International) which provided decision-analysis-based management consulting. This group subsequently spawned many other decision analysis consulting firms including Strategic Decisions Group (founded by Howard, Matheson, Carl Spetzler, and others from SRI) and Applied Decision Analysis.

3. The Decision Analysis Department at *Management Science*

In the early years of *Management Science*, there were no formal editorial departments focusing on particular subject areas and the journal published relatively little decision analysis research. When *Management Science* first created departments in 1969, there was a decision theory department with H. O. Hartley serving as department editor (DE). In March of 1970, the name of the department was changed to decision analysis, which reflected the new terminology advocated by Howard (1966, 1968) and Raiffa (1968). Howard served as the DE from 1970 until 1981; Robert Winkler then served as DE from 1981 until 1989. In 1989, the decision analysis department adopted a

Figure 1   History of Department Editors for *Management Science*'s Decision Analysis Department

1970	1975	1980	1985	1990	1995	2000	2005

			Decision Analysis -Theory and Methodology		
		I. H. Lavalle	R. T. Clemen	R. F. Nau	J. E. Smith
Decision Analysis					
R. A. Howard	R. L. Winkler		Decision Analysis - Empirical and Behavioral		
		G. W. Fischer	L. R. Keller	M. Weber	D. von Winterfeldt

two-DE editorial structure that remains in place today. In the current structure, one DE focuses on theoretical and methodological research and the other focuses on behavioral or empirical research. While behavioral work on decision making had previously been accepted by the decision analysis department occasionally, the new structure explicitly consolidated normative, descriptive, and prescriptive research in decision analysis at *Management Science*. Prior to 1989, much of the descriptive research on decision making appeared in what was then called the organization analysis, performance, and design department, which now focuses more on organization theory and strategy. Similarly, work in game theory and forecasting and other aspects of decision analysis was handled by a department called planning, forecasting and applied game theory (or variations thereof), which was closed in 1990. The history of DEs for the decision analysis department is summarized in Figure 1.

As an interdisciplinary field, decision analysis research appears in a variety of different contexts and different academic journals. *Operations Research* and the new INFORMS journal *Decision Analysis* are closest to *Management Science* in their perspective on decision analysis research, but with more of a focus on applications. *Interfaces* publishes applications of OR/MS and has included many decision analysis applications papers.[3] In addition, many other high-quality journals regularly publish decision analysis research: *Organizational Behavior and Human Decision Processes, The Journal of Behavioral Decision Making, The Journal of Risk and Uncertainty, Theory and Decision, The Journal of Multi-Criteria Decision Analysis*, as well as journals focusing on particular problem areas, like *Risk Analysis* or *Medical Decision Making*. Many psychology, economics, and statistics journals also publish articles on decision making and decision theory.

Despite the many available outlets, *Management Science* has arguably been the top journal for decision analysis research. Since 1990, INFORMS' Decision Analysis Society (DAS) has given an annual publication award recognizing the best decision analysis publication appearing in a given year; the publication may be a book or an article appearing in any

journal. Papers appearing in *Management Science* have received this award six times; three times the award has gone to books and five times to papers in other journals, with *Operations Research* (two awards) being the only other journal with more than one paper winning the DAS publication award. (A list of publication award winners may be found at www.informs.org.)

The number of decision analysis papers published in *Management Science* over time is plotted in Figures 2a and 2b, with the two plots corresponding to two different methods of counting. Figure 2a shows Hopp's (2004) counts, which were prepared by having a Ph.D. student review each article and place it in a category corresponding to one of *Management Science*'s current departments. Figure 2b was prepared using the online journal archive JSTOR to find all *Management Science* articles that contained the words "decision analysis." The JSTOR database includes all *Management Science* articles from 1954–1999. The points in the figures indicate the actual counts and the solid lines represent five-year centered moving averages of these counts. The differences in counts are easy to explain: Many articles on decision analysis may not use this terminology, particularly before the late 1960s when this term was popularized. Conversely, not all articles that mention "decision analysis" are primarily about decision analysis. However, with either method of counting, the same general picture emerges: Decision analysis research in *Management Science* grew substantially from the late-1960s into the mid-1980s and has declined somewhat since then. We will return to consider these trends in the concluding section (§5) after discussing some specific examples of decision analysis research published in the journal.

4.   Decision Analysis Research in *Management Science*

In this section we survey some central themes in decision analysis as they have been developed in *Management Science*. As indicated in the introduction, our goal in this discussion is not to provide a comprehensive review or history of these topics or of everything that has appeared in *Management Science* but to highlight some of the interesting research that has

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[3] For a detailed discussion on publications of decision analysis applications, see Corner and Kirkwood (1991) and Keefer et al. (2004).

Figure 2   The Number of Decision Analysis Articles in *Management Science*
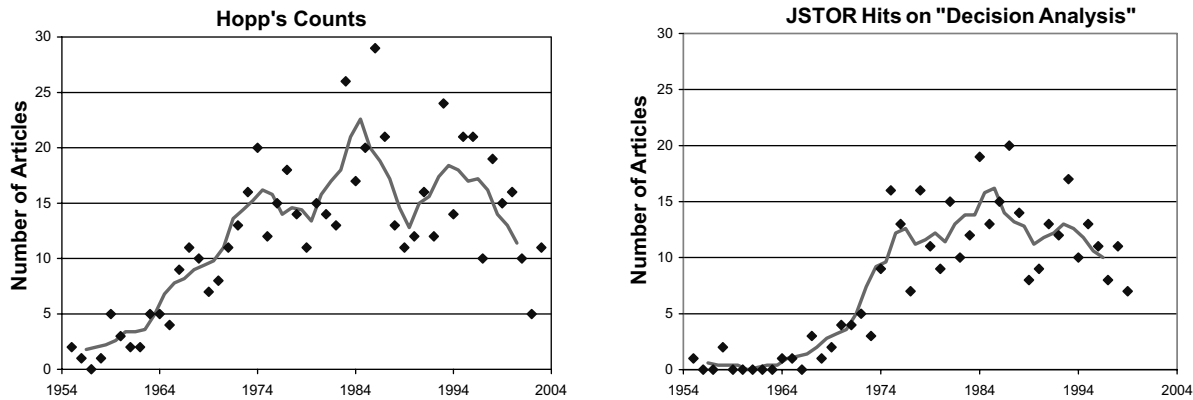
Note. The lines represent five-year centered moving averages of the counts.

appeared in the journal. We highlight developments in the assessment of probabilities in §4.1, the assessment of utilities in §4.2, and game theory and competitive decision making in §4.3.

4.1.   Developments in Probability Assessment
As discussed in §2, one of the foundations of decision analysis is the use of personal or subjective probabilities. This approach is Bayesian in that probabilities are interpreted as measures of an individual's beliefs rather than long-run frequencies to be estimated from data. One of the central challenges of decision analysis is reliably assessing probabilities from experts, taking into account the psychological heuristics that experts use in forming these judgments and the potential for biases.

Early work on probability assessment was carried out by researchers in academia and consulting practice. Spetzler and Staël von Holstein (1975*MS*) provided an overview of the psychological and practical issues associated with probability assessment, as understood at that time. They describe a probability assessment protocol that helps experts express their knowledge clearly in probabilistic terms, while avoiding judgmental biases as much as possible. Though psychological research since the 1970s has greatly improved our understanding of heuristics and biases, Spetzler and Staël von Holstein's protocols and variations thereof are still widely used today. Wallsten and Budescu (1983*MS*) provided a comprehensive review of psychological and psychometric work related to probability assessment. More recent work on probability assessment in *Management Science* has focused on decomposition (Ravinder et al. 1988*MS*, Howard 1989*MS*) and the assessment of dependence relations among uncertainties (Moskowitz and Sarin 1983*MS*, Clemen et al. 2000*MS*).

A related issue concerns the development of techniques for evaluating probabilistic forecasts and/or

providing incentives for experts to provide their best forecasts. This literature on "scoring rules" began in the domain of meteorology and statistics (see Murphy and Winkler 1970 for an early review) but was subsequently developed in *Management Science*. Matheson and Winkler (1976*MS*), Sarin and Winkler (1980*MS*), and Winkler (1994*MS*) further developed scoring rules, and Winkler and Poses (1993*MS*) evaluated the accuracy of physicians' assessments of survival probabilities. In these studies, researchers distinguish between the "calibration" of an expert (the correspondence between the stated probabilities and actual observed frequencies) and the "resolution" of the expert (the ability to distinguish and assign different probabilities to different cases). It is possible for an expert to have good resolution and be poorly calibrated or vice versa. Harrison (1977*MS*) showed how uncertainty about the calibration of an expert can cause significant practical difficulties in decision analysis by introducing dependence among events that would otherwise (i.e., without miscalibration) be independent.

In many applications of decision analysis, the stakes are sufficiently large that a decision maker will seek the opinions of several experts rather than rely solely on the judgment of a single expert or on his or her own expertise. This then raises the question of how to combine or aggregate these expert opinions to form a consensus distribution to be used in the decision model. *Management Science* has been a central outlet for research in this area. Winkler (1968*MS*) was one of the first to consider the problem and compared weighted averages and a Bayesian approach based on the use of natural conjugate distributions. Morris (1974*MS*, 1977*MS*) suggested the use of a Bayesian approach in which the decision maker begins with a prior probability distribution on the event or quantity of interest, and treats the assessments provided by the experts as observations that lead the decision maker

to update his prior probabilities using Bayes' rule. This is a normatively sound approach, but requires some very difficult assessments: The decision maker must specify the probability that an expert will state each possible response, conditional on the true state of the event or value of the uncertain quantity. With multiple experts, the decision maker must specify a joint conditional distribution for all expert responses simultaneously. Morris (1977*MS*, p. 687) described the construction of general models of expert dependence "one of the future challenges in the field of expert modeling."

Subsequent work on expert aggregation in *Management Science* has focused on developing models for structuring and simplifying the assessments required in this Bayesian approach. Winkler (1981*MS*) developed a model that considers dependence in the experts' errors, the differences between the expert's stated mean and the observed outcome. If the errors have a multivariate normal distribution with known covariance, the consensus distribution is a normal distribution with a mean that is a linear combination of the experts' individual means with weights reflecting the accuracy of and dependence among the experts. This result is in contrast with the commonly used "linear opinion pools" (see, e.g., Bacharach 1975*MS*, DeGroot and Mortera 1991*MS*), which result in a consensus distribution with a density that is a weighted average of the individual experts' densities; for example, a mixture of normal distributions. Clemen (1987*MS*) considered models of dependent experts in which the experts share some information and showed how additional information provided to the experts may confound the expert opinions and actually make a decision maker worse off.

Morris (1983*MS*) proposed an axiomatic approach to the expert combination problem and generated a great deal of discussion and controversy, including comments by Lindley (1986*MS*), Schervish (1986*MS*), Clemen (1986*MS*), French (1986*MS*), and a rejoinder by Morris (1986*MS*) as well as a summarizing discussion by Winkler (1986*MS*). The axiomatic approach was intended to complement the Bayesian approach developed in Morris (1974*MS*, 1977*MS*) by positing a set of regularity conditions to simplify the modeling problem and reduce the assessment burden. Morris' axioms assumed that (a) the consensus distribution should not depend on who observes a piece of data if there is agreement on the likelihood function and (b) a uniform prior distribution from a well-calibrated expert should not change the decision maker's opinion. Given these assumptions, Morris concluded that if an expert is well-calibrated, the decision maker has a (noninformative) uniform prior, and the decision maker's and expert's opinions are independent, then it is appropriate to elicit probabilities from a single expert and use them directly in a decision analysis, as "most decision makers (and decision analysts) do all the time without thinking" (Morris 1986*MS*, p. 322). Morris called these conditions "quite restrictive" suggesting that this common practice is inappropriate.

One point of recurring debate in this literature is the unanimity principle: Given two experts who agree on the probability of an event (for example, two meteorologists say that the probability of rain on a given day is 0.55), should the consensus probability match this common probability or should it be a different number? While there was clearly some disagreement about the interpretation and usefulness of Morris's axioms, Morris and the others all seemed to agree that one should approach the expert combination problem through the use of Bayesian models and the answer to questions like this unanimity question should depend on the specifics of a given problem.

While it is easy to say that the Bayesian modeling approach represents the "solution" to the expert combination problem in principle, in practice there remain many complex modeling challenges and questions about the effectiveness of different combination mechanisms. Clemen and Winkler (1990*MS*) described an empirical test of the unanimity principle and its generalization into the compromise principle (consensus probabilities should lie between differing probability forecasts) and compared the performance of several Bayesian models using a large dataset of probability of precipitation forecasts. Their results illustrated the importance of capturing dependence among the expert forecasts when combining forecasts.

The expert combination problem remains an interesting and active area. Clemen and Winkler (1993*MS*) described a flexible influence-diagram-based modeling approach to this problem, and Myung et al. (1996*MS*) described a maximum entropy approach. Hora (2004*MS*) presents a study of calibration in linear combination schemes.

Several application papers have focused on probability assessment. North and Stengel's (1982*MS*) analysis of funding alternatives for the U.S. Department of Energy's magnetic fusion energy research program focused on the elicitation of probabilities for research outcomes, some of them unforeseeable and far in the future. Keeney et al. (1984*MS*) described the assessment of uncertainties about potential adverse health effects associated with carbon monoxide emissions. This involved 14 experts and complex dose-response modeling. Keefer (1991*MS*) described a model used to determine bids for offshore oil and gas leases where the dependence among the values of the leases was a key element of the model.

4.2. Developments in Utility Modeling and Assessment

The early developments in utility theory were summarized in *Management Science* in 1968 in a review article by Peter Fishburn (1968*MS*). While the early focus had been on utilities for arbitrary consequences and for money (as in Pratt 1964), in the 1970s attention focused on the assessment of utilities and on multiattribute utility functions. The work on multiattribute utility functions is discussed in detail in Keeney and Raiffa's classic book *Decisions with Multiple Objectives* (1976) and is built on contributions from many authors in many fields including research published in *Management Science* (Fishburn 1965*MS*, 1967*MS*; Keeney 1972*MS*). Later, Bell (1979*MS*) described techniques for assessing multiattribute utility functions that cannot be decomposed into an additive or multilinear combination of univariate utility functions. von Winterfeldt and Edwards (1986) provided an overview of both single and multiattribute utility assessment as well as a review of the related behavioral literature. Huber (1974*MS*) provided an early review of applications of multiattribute utility. Later applications of multiattribute utility theory include Bodily (1978*MS*), Crawford et al. (1978*MS*), Ford et al. (1979*MS*), Golabi et al. (1981*MS*), Gregory and Keeney (1994*MS*), and Parnell et al. (1998*MS*).

A great deal of behavioral decision research related to utility assessment has appeared in *Management Science*. Farquhar (1984*MS*) provided an early review of utility assessment techniques. Hershey et al. (1982*MS*) and Hershey and Schoemaker (1985*MS*) explored biases in the two main techniques for assessing utilities. In the probability equivalence (PE) method, one asks what probability in a binary gamble would make the decision maker indifferent between taking the gamble and receiving a fixed amount for certain. In the certainty equivalence (CE) method, one fixes the probabilities and varies the certain amount instead. If the decision maker or experimental subject truly followed expected utility theory, the two assessment methods would yield identical utilities. However, the experiments of Hershey et al. demonstrated that the assessed utilities depends on the probabilities used in the assessment gambles, and that the CE method generally yields greater risk taking behavior than the PE method. Such behavior is consistent with subjects inappropriately weighting probabilities as suggested by Kahneman and Tversky (1979). To reduce the impact of this probability weighting, McCord and de Neufville (1986*MS*) proposed asking subjects to compare two gambles (rather than a gamble and sure thing) and showed that their method produced utilities that depend less on the probabilities used in the gambles. Johnson and Schkade (1989*MS*) studied these biases in more detail and, later, Wakker and

Deneffe (1996*MS*) proposed a more complex trade-off assessment procedure that uses the same probabilities in the two gambles and thus circumvents problems associated with subjects inappropriately weighting probabilities. More recently, Bleichrodt et al. (2001*MS*) used descriptive models to correct biases in utility assessments.

Behavioral decision research has also affected how decision analysts define and structure utility functions; here we emphasize results on proxy attributes and splitting effects. A proxy attribute is an indirect measure of the degree to which some more fundamental, harder to measure objective is obtained. For example, response time measures are often used as proxies for more fundamental measures of emergency system performance, like lives saved or fire damage averted. Keeney and Raiffa (1976) suggested that utilities assessed for proxy attributes may be biased because it is difficult for decision makers to fully comprehend the relationship between the proxy attribute and the corresponding fundamental attributes. Fischer et al. (1987*MS*) tested this hypothesis by comparing assessments in a pollution control setting. In one treatment, subjects provided utilities for "pollution control costs" and the fundamental attribute "pollution-related illnesses." In another treatment, subjects provided utilities for "control costs" and the proxy attribute "pollution emission levels," with a mathematical model relating emission levels to illnesses. Fischer et al. showed that subjects consistently overweighted the proxy attribute "emission levels" as compared to the implied weight for this proxy attribute derived from preferences for the fundamental attribute "illnesses" and the model relating the proxy and fundamental objectives. This research suggests that analysts should focus utility assessments on the fundamental attributes rather than proxies, but at the cost of increasing the modeling burden to capture the sometimes complex and controversial relationship between proxy and fundamental attributes.

Weber et al. (1988) studied how weights in multiattribute utility assessments change depending on the level of detail in a hierarchical multiattribute utility function. For example, they considered a multiattribute job-selection model and compared the weights associated with the attribute "job security" when it was treated as a single objective and when the same attribute is decomposed into two component elements, "stability of the firm" and "personal job security." Weber et al. found that the level of detail used in the specification greatly impacted the weight assigned to the attribute: Attributes that are decomposed in more detail received more weight than the same attribute with a less detailed decomposition. These results suggest that analysts need to take

great care in defining a value hierarchy for utility functions.

While most of the work in utility has considered the perspective of a single decision maker, there has also been a significant amount of research in *Management Science* on normative models for group decision making. Arrow's famous "Impossibility Theorem" (1951b) showed that given arbitrary orderings of consequences by individuals in a group, there is no procedure for generating a group order that satisfies a set of seemingly reasonable behavioral assumptions. However, Keeney (1976*MS*) showed that if we begin with cardinal utilities (rather than ordinal utilities), we can easily find a group utility function that satisfies a suitably reinterpreted version of Arrow's reasonable assumptions. Under uncertainty and with expected utilities for individuals and the group, this group utility is a weighted sum of the individual utilities with the weights reflecting the decision maker's (a social planner or benevolent dictator) trade-offs between the utilities of the group members; this had been shown in Harsanyi (1955). Keeney and Kirkwood (1975*MS*) and Dyer and Sarin (1979*MS*) studied more general forms of group preference functions, the former focusing on decisions under uncertainty and the latter using "strength of preference" notions to study group decisions under certainty. While these models considered aggregating individual utilities, Eliashberg and Winkler (1981*MS*) considered the case where each individual's utility function itself depends on what other members of the group receive.

A major recurring theme in the literature on group decision making is concern with the equity and fairness of the group decision and the allocation of the costs, benefits, and risks associated with the decision. Keeney (1980*MS*) developed the concept of an equitable distribution of risk and identified utility functions consistent with basic attitudes toward equity. Harvey (1985*MS*) studied preferences for equity in more detail, developing notions of "inequity neutrality" and "inequity aversion" and studying the implications for the form of the group utility function. Fishburn and Sarin (1991*MS*) focused on "dispersive equity," looking at the distribution of risks among subgroups in a population and later studied "fairness" and "envy" in social risk contexts (1994*MS*, 1997*MS*).

One of the more acrimonious debates in *Management Science* has concerned the Analytic Hierarchy Process (AHP). The AHP is a decision-making procedure originally developed by Thomas Saaty in the 1970s and described in Saaty (1980). Decision analysts have been critical of the AHP saying that it lacks a strong normative foundation and that the questions the decision maker must answer are ambiguous. While Saaty (1986*MS*) provided an axiomatic

foundation for the AHP, these axioms conflict with the axioms of expected utility theory and have met with resistance from decision analysts. Harker and Vargas (1987*MS*) provided a wide-ranging defense of the AHP countering some of these criticisms. Dyer (1990*MS*) summarized the concerns of decision analysts and responded to Harker and Vargas: Dyer's main point was that "the results produced by the [AHP] are arbitrary" (p. 254). Dyer illustrates this by presenting examples where three alternatives (*A*, *B*, and *C*) are ranked in order $B > A > C$ by the AHP. When we add a fourth alternative, *D*, that is an exact copy of *C*—meaning it has identical evaluations on all attributes—the alternatives are ranked in the order $A > B > C = D$; thus, the introduction of an irrelevant alternative causes *A* and *B* to switch orders! Dyer pointed out that the same phenomenon occurs with "near copies" as well and suggested a solution to the flaws of the AHP using techniques from multiattribute utility theory.

Saaty (1990*MS*) and Harker and Vargas (1990*MS*) both wrote replies to Dyer's article addressing many specific points and generally not accepting Dyer's comparison to the normative standard of utility theory. Reading these replies, however, it is difficult to understand whether Saaty and Harker and Vargas intend the AHP to be prescriptive or descriptive procedure. They seem to use the AHP as a prescriptive procedure in applications, but Saaty writes that:

> Utility theory is a normative process. The AHP as a descriptive theory encompasses procedures leading to outcomes as would be ranked by a normative theory. But it must go beyond to deal with outcomes not accounted for by the demanding assumptions of a normative theory. (Saaty 1990*MS*, p. 259)

Our view is that if the AHP is truly intended as a descriptive model, then one should test it to see how well it describes actual decision-making behavior. Though we know of no such tests, we are confident that the AHP would not do a very good job predicting decision-making behavior, just as the expected utility model has limited descriptive power. The appeal of the AHP as a prescriptive methodology remains matter of disagreement. While many in the decision analysis community (ourselves included) follow Dyer in believing the AHP to be fundamentally unsound, others (including Saaty, Harker, and Vargas) disagree and the AHP is still widely used in practice today.

4.3. Developments in Game Theory and Competitive Decision Making
As indicated in the historical discussion of §2, game theory and decision analysis share common foundations: The systematic formal study of game

theory and utility theory both began in earnest with the publication of von Neumann and Morgenstern's *Games and Economic Behavior* in 1944. Moreover, some of the key figures in decision analysis, notably Duncan Luce and Howard Raiffa, began their careers working in game theory (see, e.g., Luce and Raiffa 1957). Since that time, however, the interests of decision analysts and game theorists seem to have diverged, but not until after some significant contributions to game theory appeared in *Management Science*.

The most prominent game theory publication in *Management Science* is the three-part series by John Harsanyi published in 1967 and 1968 (1967*MS*, 1968a*MS*, 1968b*MS*), which was the basis for Harsanyi's Nobel Prize in Economics in 1994. Consistent with the decision analyst's focus on decision making under uncertainty, Harsanyi studied games of "incomplete information" in which some or all players lack full information about some essential features of the game, including perhaps knowledge about the other players' payoff function or available actions. As Bayesians, each player assigns subjective probabilities over these uncertain features of the game. Moreover, Player 1 would be uncertain about the probabilities that Player 2 assigns and, as a rational Bayesian, would assign probabilities on Player 2's probabilities. Player 2 would do likewise. Player 1 then has to assign another level of probabilities on the probabilities that Player 2 would assign to the probabilities that Player 1 assigns. Player 2 would do likewise and the sequence continues rationally but impractically ad infinitum.

Harsanyi's contribution was the development of a framework that logically models games of incomplete information but avoids this infinite regress and reduces the game to a game of complete information with uncertainty about the "types" of the players involved in the game. The "type" of a player contains information about all players' first-order probabilities and payoffs. Harsanyi assumed that there is an exogenously given joint probability distribution on the types of all players in the game that is interpreted as a "common prior." At the beginning of the game, each player knows his own type but not that of his opponents. The players then update their probabilities on the other players' types using Bayes' Theorem based on this and whatever other information revealed during the course of the game. Harsanyi's first paper in the three-part series (Harsanyi 1967*MS*) showed that, given the consistency requirement of a common prior on types, any game of incomplete information can be represented in this "flattened" form. In the second paper (Harsanyi 1968a*MS*), Harsanyi showed that the Nash equilibria in the flattened game, which Harsanyi called a Bayesian equilibria, correspond to the equilibria of the original game. In the third paper (Harsanyi

1968b*MS*), he studied properties of the distribution on types in the model. Harsanyi's Nobel Prize lecture (Harsanyi 1995) provides an overview of this work.

Games of incomplete information have been used to analyze negotiation, competitive bidding, social choice, the signaling roles of education and advertising, as well as many other economic phenomena. Nevertheless, game theory has its critics, including many in the decision analysis community. Howard Raiffa (1982, p. 2), for example, wrote that game theory "deals only with the way in which ultrasmart, all-knowing people *should* behave in competitive situations and has little to say to Mr. X as he confronts the morass of his problem." Raiffa, in his study of negotiations, instead preferred to focus on an "asymmetrically prescriptive/descriptive" perspective in which one seeks to help a party compete against an opponent whose behavior is not necessarily assumed to be rational, or alternatively helps many parties (not necessarily rational) reach a joint decision. This perspective has been adopted in much recent work in negotiation analysis; in particular, see Sebenius (1992*MS*). Pratt and Zeckhauser (1990*MS*) provided a wonderful example of this kind of analysis describing the development and use of a formal, decentralized division procedure to fairly and efficiently divide a set of valuable silver heirlooms among beneficiaries of an estate.

An important paper in the debate on game theory is Kadane and Larkey's "Subjective Probabilities and the Theory of Games" (1982b*MS*). Kadane and Larkey argued that a Bayesian decision maker need only take into account his first-order beliefs about his opponent's play at the time he chooses actions, without getting caught up in the infinite regress or strong assumptions about the rationality of the opponent. They wrote

> It is true that a subjectivist Bayesian will have an opinion not only about his opponent's behavior, but also about his opponent's belief about his own behavior, his opponent's belief about his belief about his opponent's behavior, etc. (He also has opinions about the phase of the moon, tomorrow's weather and the winner of the next Superbowl.) However, in a single-play game, all aspects of his opinion except his opinion about his opponent's behavior are irrelevant, and can be ignored in the analysis by integrating them out of the joint opinion." (Kadane and Larkey 1982b*MS*, p. 116)

Thus, in Kadane and Larkey's view, there is no need for the hierarchy of beliefs or special structures or rationality assumptions. The various solution concepts of game theory provide the basis for assigning particular prior distributions, but are not necessary

for the solution of the decision problem. Harsanyi (1982*MS*) wrote in a comment

> Kadane and Larkey oppose any use of normative solution concepts and oppose imposing any rationality criteria on players' choice of subjective probabilities. They do not seem to realize that their approach would amount to *throwing away essential information*, viz., the assumption (even in cases where this is a realistic assumption) that the players will act rationally and will also *expect* each other to act rationally. Indeed, their approach would trivialize game theory by depriving it of its most interesting problem, that of how to translate the intuitive assumption of mutually expected rationality into mathematically precise behavioral terms (solution concepts). (p. 121)
>
> Kadane and Larkey have not proposed any viable alternative to this approach. All they have proposed is to trivialize game theory by rejecting the basic intellectual problem and to replace it by the uninformative statement that every player should maximize his expected utility in terms of his subjective probabilities without giving him the slightest hint of how to choose these subjective probabilities in a rational manner. (p. 123)

In their reply to Harsanyi, Kadane and Larkey (1982a*MS*) suggested the use of what we now call an asymmetrically prescriptive/descriptive mode of analysis that uses an "empirically supported psychological theory making at least probabilistic predictions about the strategies people are likely to use, given the nature of the game and their own psychological makeup" (p. 124). In this response, they paraphrased remarks in Harsanyi's reply, but while Harsanyi suggested the need for such a psychological theory for playing against "actually or potentially irrational opponents," Kadane and Larkey argued that most opponents are in fact "actually or potentially irrational" and such a psychological theory should be central in the study of competitive decision making.

In a follow-up paper titled "The Confusion of Is and Ought in Game Theoretic Contexts" (1983*MS*), Kadane and Larkey called the 30+ years of work in game theory "cumulatively useless" in that it provides "so little of value in instructing people on how they *should* behave in conflict situations and in predicting how they *do* behave in conflict situations" (p. 1370). This prompted a comment by another prominent game theorist, Martin Shubik (1983*MS*), who acknowledged that Kadane and Larkey raise good questions. However, like Harsanyi, Shubik said that "replacing an *n*-person game by *n* parallel one-person games with subjective probability updating black boxes solves no problems, it slurs over them" (p. 1383).

In a plenary address given at the 1987 Institute of Management Sciences meeting called "What Is an Application and When Is a Theory a Waste of Time?" (Shubik 1987*MS*), Shubik reconsidered the usefulness of game theory and distinguished between the "high-church" version of game theory concerned with formal, mathematical structures and analysis; "low-church" applications of basic concepts to specific problems; and the "conversational" game theory consisting of advice, suggestions and counsel about how to think strategically. Shubik acknowledged that low-church applications of game theory have been of "some, but nevertheless relatively modest worth" but that the conversational version of game theory is of considerable worth (p. 1516). However, he noted that

> Without high church game theory, the concepts, illustrations and stories of conversational game theory would hardly exist and certainly would not have a sound intellectual basis. Without conversational and high church game theory, sponsorship for low church game theory would hardly exist.
>
> Application is not just calculation and specific problem solving. It is also concept clarification, education, and changing modes of thought. (p. 1517)

Rothkopf and Harstad (1994*MS*) expressed similar sentiments in their review of the use of game-theoretic models of auctions in actual competitive bidding situations.

To us, it appears that most decision analysis researchers have come to prefer the more practically oriented asymmetrically prescriptive/descriptive perspective on competitive decision making advocated by Raiffa, Kadane and Larkey, and others to the ultrarational normative/normative perspective of "high-church" game theory. While "low-church" applications of game theory have become quite popular in some areas of *Management Science*, particularly in supply chain analysis (see, e.g., Cachon and Zipkin 1999*MS*), the decision analysis department has not published much in the way of "high-church" game theory in recent years. We have however begun to see considerable activity in "behavioral game theory" where one studies the actual behavior of participants in various forms of games (see, e.g., Bolton et al. 2003*MS*). Colin Camerer's new book, *Behavioral Game Theory* (Camerer 2003), provides a comprehensive review of the current state of the art in this area.

5. Concluding and Looking Ahead
As we reviewed the decision analysis articles that have appeared in *Management Science*, we have been impressed with the depth, quality, and sheer volume of decision analysis research that has appeared in the journal. In highlighting a few topics and articles

here, we have omitted many important articles that appeared in *Management Science* but did not fit clearly into the research streams that we chose to emphasize, as well as many important contributions published elsewhere. Our choice of topics was intended to highlight how prescriptive decision analysis research builds on the foundations of normative and descriptive research on decision making.

While the number of decision analysis papers appearing in *Management Science* increased dramatically from the 1960s through the mid-1980s, we have witnessed a general decline since that time. We believe that this decline in papers does not reflect a decline in the relevance of decision analysis or in related research. Interest in behavioral decision-making research in particular is growing, although it is less likely to use the term decision analysis and less likely to have the managerial and prescriptive orientation of *Management Science*. *The Journal of Behavioral Decision Making* and *Organization Behavior and Human Decision Processes* now publish a great deal of behavioral decision-making research and top journals in economics and finance now regularly publish work in experimental economics and behavioral finance. Daniel Kahneman won the Nobel Prize in Economics in 2002 for his research, much of it with the late Amos Tversky, on decision-making behavior under uncertainty. The prize was shared with Vernon Smith who was cited for his laboratory experiments studying behavior of humans interacting in different market settings. As editors at *Management Science*, we would like to see more work relating these new developments in behavioral decision making back to the prescriptive modeling of decision analysis.

Prescriptive decision analysis research is also growing and appearing in a variety of new settings. As part of a recent study on the viability of the new journal *Decision Analysis*, Don Kleinmuntz conducted a literature search that looked for articles published from 1994 to 1998 that used "decision analysis" in the title, subject key words, or abstract (Keller and Kleinmuntz 1998). The search found a total of 811 articles in 369 journals. Strikingly, over 60% of the articles identified were published in medical journals and some 220 different medical journals were represented. The other 149 journals represented fields such as environmental risk management, engineering, artificial intelligence, psychology, as well as management.

Decision analysis has clearly been recognized as an important tool for the evaluation of major decisions in the public sector, particularly in the healthcare arena. Although a handful of consulting firms have demonstrated the value of decision analysis to corporate clients in evaluating research and development projects, oil and gas exploration opportunities, and other areas, the use of decision analysis

methods is not yet widespread in corporations. To have a greater impact on corporate decision making, we believe that decision analysis researchers must build on and pay more attention to the principles of corporate finance and the theory of financial markets. *Management Science* has published some papers that integrate finance and decision analysis. For example, Wilson (1969*MS*) used probabilistic models of investments and financing decisions to develop conditions for accepting or rejecting individual projects without considering the entire portfolio of projects. Smith and Nau (1995*MS*) considered the evaluation of projects where some project risks can be hedged by trading marketed securities; their methods integrate "risk-neutral" valuation techniques used to value derivative securities with traditional, decision-analytic certainty-equivalent-based notions of valuation. However, despite these advances, at present there is no consensus on how firms should evaluate risky cash flows and much research remains to be done.

As we have seen, *Management Science* has played a central role in the development of the field of decision analysis over the last 50 years, particularly during the last 35 years. We hope that *Management Science* will continue to play a central role in future, along with INFORMS' new *Decision Analysis* journal. In particular, we hope that decision researchers will continue to do research with the prescriptive goal of improving managerial decision making and that they will continue to write up their best work for the broad audience at *Management Science*.

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