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Technology Adoption with Uncertain Future Costs and Quality

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In this paper we study the impact of uncertainty about future innovations in quality and costs on consumers' technology adoption decisions. We model the uncertainty in the technology's quality and costs as a Markov process and consider three models of the adoption decision. The first model assumes that consumers do a simple net present value (NPV) analysis that compares the NPV of adopting to that of not adopting, without considering the possibility of waiting. The second model is a stochastic dynamic program that considers the possibility of waiting and views the adoption decision as a one-time event, i.e., the consumer will only make a single purchase, the only question is when. The third model allows repeat purchases so the consumer may "upgrade" by purchasing new versions of the technology whenever it suits her.

We study structural properties of these models, e.g., the following: What changes in qualities and costs will make the consumer better off? What changes will encourage adoption? We will see that the simple NPV and single-purchase model have many intuitive properties: with the right notion of improvements and reasonable assumptions about the technology changes, we find that improvements in the technology make the consumer better off and encourage adoption. Here improvements are defined using a partial order on quality and cost pairs. The results are more complicated in the repeat-purchase model. Under the same conditions on technology changes, technology improvements will make the consumer better off. However, except for special cases of transitions, these improvements may make the consumer better off and *discourage* adoption.

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1. Introduction

Technology adoption decisions are notoriously difficult, and history is full of examples where firms and consumers have been "slow" to adopt a new technology. For example, Schumpeter (1934, p. 15) wrote: "we see all around us in real life faulty ropes instead of steel hawsers, defective draught animals instead of show breeds, the most primitive hand labor instead of perfect machines, a clumsy money economy instead of a cheque circulation, and so forth." Modern consumers and firms are similarly vexed by technology adoption decisions: Should I buy a new computer (hybrid car, video camera, cell phone, television, MP3 player, version of software, ...) now or wait for future improvements and/or cost reductions? Should an electric utility meet increasing demand by building a new power plant now or wait for capital costs to decrease, efficiencies to improve, or for regulatory uncertainty to be resolved?

There is a growing theoretical literature on technology adoption decisions that formally studies the impact of uncertainty about future technological improvements on adoption decisions. In this literature, we can distinguish three prototypical models of adoption decisions. The first model assumes that the consumer simply compares the net present value (NPV) of the lifetime costs and benefits associated with adopting the technology to the NPV associated with not adopting, without considering the possibility of waiting or the possibility of upgrading in the future. This *simple NPV model* is commonly used in practice and serves as a benchmark for comparison: adoption might be considered "slow" if consumers fail to adopt technologies when the lifetime NPV associated with adoption exceeds the lifetime NPV associated with not adopting.

The second prototypical model is a stochastic dynamic program that considers uncertainty about how the technology will evolve over time and explicitly considers the option of waiting to adopt. In this *single-purchase model*, the consumer can exercise the option to adopt the technology whenever she wants. As is often noted in the real options literature (see, e.g., Dixit and Pindyck 1994), in this framework it may be optimal to be "slow" and wait for the possibility of future improvements, even though a simple NPV analysis suggests adopting. For example, Farzin et al. (1998) view technology adoption as a real option and consider a continuous-time model with uncertainty about the timing and magnitude of technological improvements. They show that this new technology will be adopted as soon as its value exceeds a certain threshold that is higher than the

NPV threshold. Doraszelski (2001, 2004) clarifies and generalizes Farzin et al. (1998) and studies the behavior of adoption policies in numerical experiments.

The third prototypical model is a more sophisticated stochastic dynamic program that considers uncertainty about how the technology will evolve but, more realistically, considers the possibility of repeat purchases. In this *repeat-purchase model*, the consumer may adopt the current version of the technology now and upgrade by buying again at some point in the future. In a classic paper in this vein, Balcer and Lippman (1984) consider a model where the timing and magnitude of future improvements is uncertain and the consumer may repeatedly purchase the technology. In their model, they find that it is optimal to adopt the new technology whenever the lag between the technology available and the technology owned exceeds a threshold. Kornish (1999) clarifies some results in Balcer and Lippman (1984), and Cho and McCardle (2009) generalize Balcer and Lippman's model to consider adoption decisions for two technologies where it may be cheaper (or more expensive) to adopt multiple technologies simultaneously.

In this paper, we consider richer models of technology change and study the impact of uncertainty about future improvements in these three prototypical technology adoption models. The model of technology change is richer than those in the literature because (i) we consider uncertainty in future adoption costs as well as uncertainty about the quality of future technologies and (ii) we consider more general models of quality improvements. Whereas the previous literature assumes that quality improvements come in the form of random increments that are independent of the current quality level, we allow the cost-quality transitions to follow a more general Markov process.

These generalizations enhance the realism of the model and lead to more-nuanced results. We show that we can obtain results about the structure of the value function and the optimal policies by defining partial orders on technology quality and cost pairs and using generalized stochastic dominance arguments. The value functions and optimal policies for the simple NPV and single-purchase model have a nice monotonic structure. In the simple NPV model, any change in cost and/or quality that increases the lifetime NPV associated with adoption (obviously) makes the consumer better off and encourages adoption. In the single-purchase model, not all increases in the lifetime NPV make the consumer better off, but we can characterize a set of cost and quality improvements—which we call "clear improvements"—that make the consumer better off and encourage adoption, given certain reasonable conditions on the technology transitions. These clear improvements are a subset of those changes that increase the lifetime NPV associated with adoption: with the possibility of delayed adoption, it is not clear when (or if) the technology will be adopted and, consequently, some improvements in lifetime NPV may not be appreciated by a consumer who is waiting.

The results for the repeat-purchase model are more delicate. Under the same reasonable conditions as the single-purchase model, we can show that clear improvements make the consumer better off. However, we cannot establish analogous monotonicity results for the policies. Although we might expect clear improvements to encourage adoption, in fact, some clear improvements in the technology may make the consumer better off by encouraging the consumer to switch from adopting to not adopting in the current period. As we will illustrate, improvements in the technology may make it unnecessary to upgrade in the future and this, in turn, can lead to waiting in the current period. However, if we assume that the cost-quality improvements have additive increments that are independent of the current cost-quality level and assume that quality is nondecreasing over time, we can show that clear improvements in cost and quality will encourage adoption, as one might expect.

Although we focus on the impact of uncertainty about the quality and cost of future versions of the technologies, there is another stream of literature on technology adoption that instead focuses on the impact of uncertainty about the quality or profitability of the current technology. These models consider the technology adoption decision to be a one-time event, and the technology typically does not change over time; see, e.g., Jensen (1982), McCardle (1985), Lippman and McCardle (1987), Ulu and Smith (2009). Uncertainty about the current quality provides another reason consumers may be "slow" to adopt a technology: even though the consumer may estimate a positive expected NPV associated with adopting, it may still be best to wait and gather additional information to be more sure that the technology is truly profitable.

We begin in §2 by defining and comparing the adoption models we study. In §3, we study monotonicity properties of the value functions and optimal policies. In §4, we briefly study convexity properties. We offer concluding remarks in §5. A few proofs are provided in the paper; the remainder are provided in an electronic companion, along with some technical discussions. An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

2. The Models

We begin by describing the technology transition models and then describe the simple NPV, single-purchase, and repeat-purchase models. Next, we present an illustrative numerical example and some formal results comparing the different models.

2.1. Technology Transitions

Time is discrete and finite; we let $k = 0, 1, \ldots$ denote the number of periods remaining. There is a technology available in the marketplace whose cost and quality change stochastically over time. In period k, the cost to adopt the technology is c_k. The period-k quality p_k is defined as the per-period benefit the technology provides to the consumer when she owns the technology. We assume that the cost c_k is a one-time cost to acquire the technology and that, once adopted, the per-period benefit of the technology does not change over its lifetime. The available technology evolves stochastically according to a Markov process with the probability distribution for the next-period quality and cost $(\tilde{p}_{k-1}, \tilde{c}_{k-1})$ depending on the current quality and cost (p_k, c_k); the changes in $\tilde{p}_{k-1}$ and $\tilde{c}_{k-1}$ may be correlated. The consumer is initially endowed with a technology (or substitute for the technology) with quality q_k.

This model of technological change is general enough to handle a variety of different kinds of innovation processes. For example, the next-period technologies $(\tilde{p}_{k-1}, \tilde{c}_{k-1})$ could represent incremental improvements where $\tilde{p}_{k-1}$ and $\tilde{c}_{k-1}$ are drawn from values slightly above p_k and/or below c_k. Alternatively, the transitions could represent occasional breakthrough innovations or "jumps" where most of the mass for $(\tilde{p}_{k-1}, \tilde{c}_{k-1})$ is concentrated at the current value (p_k, c_k), but there is some chance of drawing a significantly higher value for $\tilde{p}_{k-1}$ and/or lower value for $\tilde{c}_{k-1}$; in this case, the arrival time for changes in technologies would be stochastic. In general, we allow costs and quality to increase or decrease over time. The framework is also general enough to capture the possibility of a technology maturing, for example, by assuming that the uncertainty in quality and costs is decreasing over time or, alternatively, as the technology improves.

We will at times consider a special case with additive transitions where

$$\begin{aligned}\tilde{p}_{k-1} &= p_k + \tilde{u}^p_{k-1}\\ \tilde{c}_{k-1} &= c_k + \tilde{u}^c_{k-1}.\end{aligned} \tag{1}$$

Here the additive increments, $\tilde{u}^p_{k-1}$ and $\tilde{u}^c_{k-1}$, are assumed to be independent of (p_k, c_k), but may be correlated with each other. In this model, anticipated improvements in quality or reductions in costs can be accommodated by assuming positive or negative expected values for the increments.

The models of Balcer and Lippman (1984) and Farzin et al. (1998) (and others discussed in the introduction) assume that quality follows this form of additive process and that costs are constant (i.e., $\tilde{u}^c_{k-1} = 0$). Moreover, they decompose the uncertainty about changes in quality into two components: first, there is uncertainty about whether there is an innovation (innovation occurs with some probability π) and second, if there is an innovation, there is uncertainty about the magnitude Z of the innovation. This can be viewed as a special case of the additive model with $\tilde{u}^p_{k-1}$ placing mass $1-\pi$ at 0 and selecting a value Z with probability π. Balcer and Lippman (1984) and Farzin et al. (1998) also assume that Z is nonnegative.[1]

2.2. Three Adoption Models

In the *simple NPV* model of the technology adoption decision, we assume that the consumer compares the NPV of adopting to the NPV of not adopting and chooses whichever leads to highest value. For $k > 0$, the value given by such a model is

$$\begin{aligned}&v^n_k(p_k, c_k, q_k)\\ &\quad = \max\begin{cases}\dfrac{1-\delta^k}{1-\delta}p_k - c_k & \text{(if she adopts)},\\[2ex] \dfrac{1-\delta^k}{1-\delta}q_k & \text{(if she does not adopt)},\end{cases}\end{aligned} \tag{2}$$

where δ $(0 \leqslant \delta < 1)$ is a discount factor. (If $\delta = 1$, we can replace $(1-\delta^k)/(1-\delta)$ with k, here and throughout the paper.) For consistency with the other models, we take $v^n_0(p_k, c_k, q_k) = 0$. The superscript n here is a mnemonic for NPV or, alternatively, "naive." This model takes into account the full lifetime benefits of the technology and weighs this against the costs, but neglects the possibility of waiting and purchasing the technology at some point in the future.

In the *single-purchase model*, the consumer can adopt or defer the adoption decision and continue with the technology that she currently owns. More specifically, if the consumer adopts the technology in period k, she pays c_k and obtains a total benefit equal to the NPV of the benefit stream less the adoption cost: $((1-\delta^k)/(1-\delta))p_k - c_k$. If the consumer does not adopt, she obtains a benefit of q_k and begins the next period endowed with the same technology. Assuming that the consumer makes decisions to maximize the discounted expected net benefit, we can write the value function recursively: we take the terminal value function to be $v^s_0(p_k, c_k, q_k) = 0$ and, for earlier periods, we take

$$\begin{aligned}&v^s_k(p_k, c_k, q_k)\\ &\quad = \max\begin{cases}\dfrac{1-\delta^k}{1-\delta}p_k - c_k, & \text{(if she adopts)},\\[2ex] q_k + \delta\mathbb{E}[v^s_{k-1}(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k, c_k] & \\ \quad \text{(if she does not adopt)}.\end{cases}\end{aligned} \tag{3}$$

Here, the superscript s is a mnemonic for "single" purchase. Thus, in this model, the consumer views the adoption decision as being like an American call option that can be exercised at any time. Adopting the technology today, however, implies that she cannot benefit from any future improvements in the technology.

In the *repeat-purchase model*, the consumer can buy a new version of the technology whenever it suits her. More specifically, if the consumer adopts the technology, she pays c_k and obtains a benefit p_k in that period and then begins the next period endowed with this new technology. If she does not adopt the technology, she receives the benefit q_k of the technology that she currently owns and begins the next period endowed with this same technology. The value function for the repeat-purchase model is then given recursively by taking the terminal value to be $v_0^r(p_k, c_k, q_k) = 0$ and, for earlier periods,

$$v_k^r(p_k, c_k, q_k) = \max \begin{cases} p_k - c_k + \delta\mathbb{E}[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k) \mid p_k, c_k] \\ \quad \text{(if she adopts)}, \\ q_k + \delta\mathbb{E}[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k, c_k] \\ \quad \text{(if she does not adopt)}. \end{cases} \tag{4}$$

The superscript r here is a mnemonic for "repeat."

Although we have formulated these models with finite horizons, one could consider their infinite-horizon limits. The properties of the value functions that we study—namely, forms of "increasing" or "convex" are examples of what Smith and McCardle (2002) call "closed, convex cone properties." As discussed there, if these properties hold for all finite horizons k, the same properties will hold in the infinite-horizon limit, provided the limiting value functions (v_∞^n, v_∞^s, and v_∞^r) exist. To ensure that these limits exist, we need to assume that the discount rate is positive (i.e., $\delta < 1$), and place some restrictions on the rewards (e.g., assume the rewards are bounded) and transitions; see, e.g., Bertsekas (1995), Lippman (1975), or Stokey and Lucas (1989) for discussion of conditions ensuring the existence of these infinite-horizon limits.

We view the three models (2)–(4) to be of increasing realism and appropriateness: a consumer should contemplate waiting in technology adoption decisions and should recognize the possibility of replacing the technology in the future. Unfortunately, as we will see, the three models are also increasingly difficult to analyze.

2.3. A Numerical Example

Figure 1 shows the three different value functions for a prototypical numerical example where the transitions approximate the additive model (1) on a finite grid. In this example, there are $k = 10$ periods remaining, and quality and costs are modeled as independent uncertainties on a grid with 84 evenly spaced values; the quality (p_k) ranges from 0 to 2.5 and the costs (c_k) range from 0 to 5. The transitions are approximately normally distributed with $\tilde{p}_{k-1}$ having mean $p_k + 0.10$ and standard deviation 0.30 and $\tilde{c}_{k-1}$ having with mean $c_k - 0.15$ and standard deviation 0.30. The transitions are approximated by rounding the quality and cost values to the nearest values of p_k and c_k on the grid. The initial quality level q_k is 0.45, and the discount factor δ is 0.99.

The value functions in Figure 1 are shown as a function of quality p_k and cost c_k. The upper surface represents the value function for the repeat-purchase model (v_k^r); the lower surfaces are the value functions for the single-purchase and simple NPV models (v_k^s and v_k^n); contour lines are shown on each surface. The colors describe the optimal policies, with the red/orange regions being the rejection regions. The vertical plane in the figure marks the points where $((1-\delta^k)/(1-\delta))p_k - c_k = ((1-\delta^k)/(1-\delta))q_k$. Technologies to the right of this plane are not cost-effective improvements over

Figure 1. Value functions for the numerical example of §2.3 (with $k = 10$ periods to go).

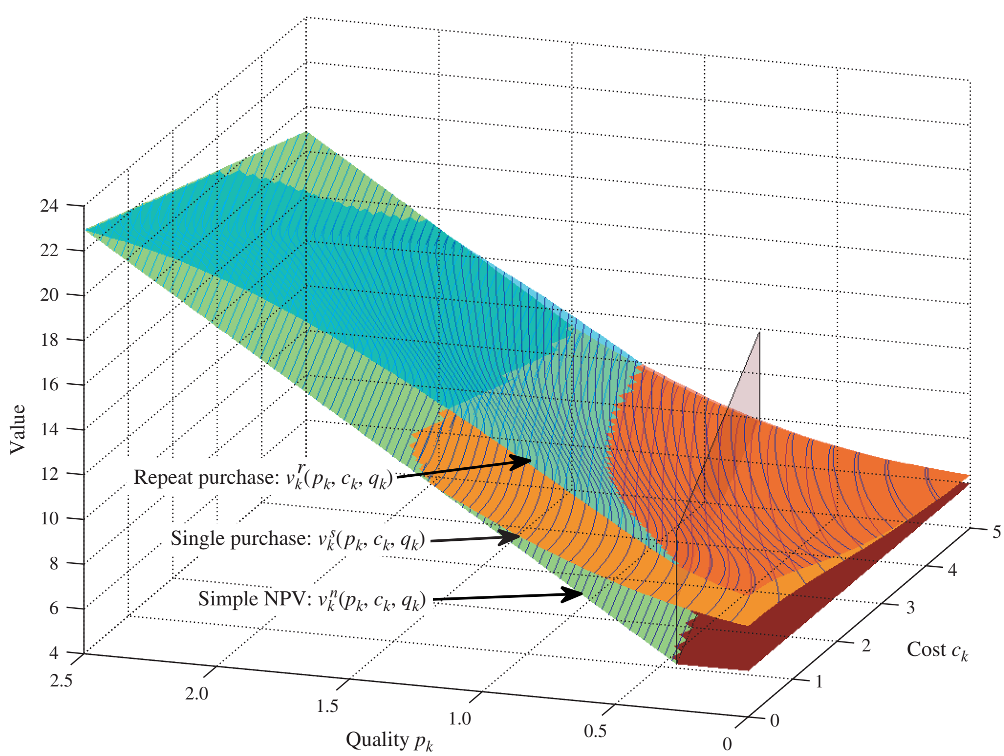
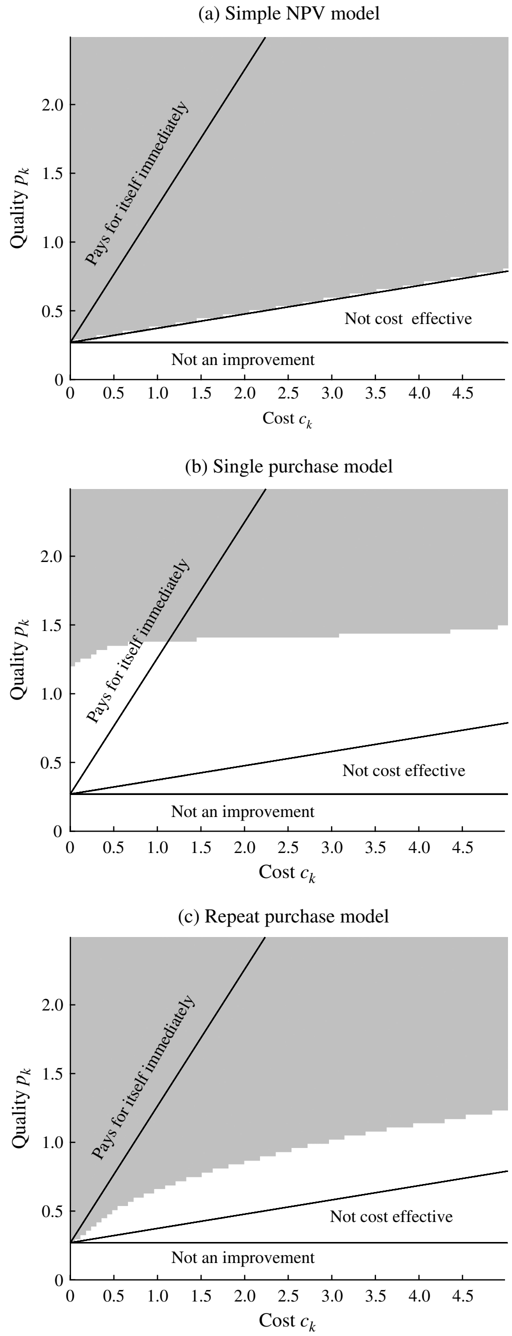


Figure 2. Adoption regions for the example of Figure 1.



the currently owned technology and would be rejected in the simple NPV model. Technologies to the left of the plane would be accepted in the simple NPV model. The adoption regions for this example are shown more explicitly as the dark regions in Figure 2.

2.4. Comparing Models

Some of the properties exhibited in the numerical example shown in Figures 1 and 2 are general properties, whereas others are specific to the example. First, it is easy to see that the value functions will always be stacked in the order shown in Figure 1. In both the single- and repeat-purchase models, the consumer could adopt the technology now and hold it for all remaining periods, if that is optimal. Similarly, in the repeat-purchase model, the consumer could adopt once (as in the single-purchase model) but could also adopt new versions of the technology later if this leads to a larger expected value.

In Figure 2, we see that technologies that are not improvements over the technology owned (i.e., $p_k \leqslant q_k$) are rejected in all three models; this will always be true if adoption is costly ($c_k \geqslant 0$). More generally, those technologies that are not cost effective (i.e., $((1-\delta^k)/(1-\delta))p_k - c_k < ((1-\delta^k)/(1-\delta))q_k$) will be rejected in the simple NPV and single-purchase models; the same is true in the repeat-purchase model if adoption is costly. In these cases, the improvement in going from q_k to p_k does not cover the cost of adoption c_k, even if the new technology would be held for all remaining periods. In the simple NPV model, the consumer will adopt all technologies that are cost effective, but in the single- and repeat-purchase models the consumer may choose to be "slow" and not adopt some of these technologies with the hope that the quality will improve or costs will decrease in the future. Similarly, it is easy to see that in the repeat-purchase model the consumer should buy any technology that pays for itself immediately (i.e., satisfies $p_k - c_k \geqslant q_k$) if adoption is costly; in this case, the consumer increases her immediate benefit and enters the next period holding a superior technology.[2] However, in the single-purchase model, the consumer may not want to adopt such a technology now because adopting now eliminates the possibility of obtaining an even better technology in the future.

In the example of Figures 1 and 2, the adoption region for the repeat-purchase model includes the adoption region for the single-purchase model. This result seems intuitive—if the consumer can only adopt once, she might wait for a better technology to be available—but does not hold in general. For instance, consider the following simple deterministic example.

Example. Consider a model with $k = 3$ periods remaining and discount factor $\delta = 1$. Suppose the consumer owns a technology with quality $q_3 = 1$ and the new technology evolves deterministically with $(p_k, c_k) = (2.8, 3), (2, 0)$, and $(3, 0)$ for $k = 3, 2, 1$, respectively. In this case, we find that it is optimal to adopt immediately in the single-purchase model. In the repeat-purchase model, it is optimal to wait in the first period and adopt in the next two periods.

If, however, the quality of the technology is certainly improving over time (i.e., $\tilde{p}_{k-1} \geqslant p_k$ almost surely), then the adoption region for the repeat-purchase model will certainly include the adoption region for the single-purchase model.

We summarize these comparisons in the following proposition.

Proposition 2.1 (Comparing Values and Policies).

1. *For all* k, p_k, c_k, *and* q_k, $v_k^n(p_k, c_k, q_k) \leqslant v_k^s(p_k, c_k, q_k) \leqslant v_k^r(p_k, c_k, q_k)$.
2. *In the simple NPV and single-purchase models, it is never optimal to adopt a technology* (p_k, c_k) *such that* $((1-\delta^k)/(1-\delta))p_k - c_k < ((1-\delta^k)/(1-\delta))q_k$. *The same result is true in the repeat-purchase model if adoption is costly* $(c_k \geqslant 0)$.
3. *In the repeat-purchase model, if adoption is costly, the consumer should adopt any technology that pays for itself immediately* (*i.e., such that* $p_k - c_k > q_k$).
4. *Suppose the quality of the technology* p_k *is nondecreasing over time, i.e.,* $\tilde{p}_{k-1} \geqslant p_k$ *almost surely for all* k. *Then, if it is optimal for the consumer to adopt a technology in the single-purchase model, it is also optimal to adopt this technology in the repeat-purchase model.*

The conclusion that the single- and repeat-purchase models may be "slow" to adopt compared to the simple NPV model and will never be "faster" than the simple NPV model is consistent with the common theme in the real options literature (see, e.g., Dixit and Pindyck 1994): in the face of uncertain future profits, it may be optimal to "wait" to invest even though a simple NPV analysis suggests investing. This is a very general result that does not require any assumptions about the form of the technology transitions. If we view the repeat-purchase model as the most appropriate model of the technology adoption decisions (as discussed in §2.2), the last part of the proposition above says that—given nondecreasing quality—the single-purchase model will generally be too "slow" to adopt compared to the standard of the repeat-purchase model.

Changing the quality of the technology that the consumer owns (q_k) has similar effects in all three models. We summarize these results in the following proposition.

Proposition 2.2 (Impact of Changing the Quality of the Technology Owned). *Let* v_k^* *denote the value function for any of the three models. In all three models*:

1. *The value function is increasing in the value of technology the consumer owns* (q_k). *That is, for all* k, p_k, c_k, *and* $q_k^1 \leqslant q_k^2$, $v_k^*(p_k, c_k, q_k^1) \leqslant v_k^*(p_k, c_k, q_k^2)$.
2. *If it is optimal to adopt technology* (p_k, c_k) *when holding a technology with quality* q_k^2, *then it is also optimal to adopt* (p_k, c_k) *when holding a technology with quality* q_k^1, *when* $q_k^1 \leqslant q_k^2$.
3. *The change in value due to changing* q_k *is bounded by the change in value if the consumer held* q_k *forever. That is, for all* k, p_k, c_k, $v_k^*(p_k, c_k, q_k) - ((1-\delta^k)/(1-\delta))q_k$ *is decreasing in* q_k.

The first two results imply that the value functions for all three models are increasing with changes in the quality of the technology-owned q_k and that the optimal policies are "decreasing" in q_k in that, if it is optimal to adopt with one q_k, it is also optimal to adopt for all lower values of q_k. The second result implies that if a consumer holding technology q_k^1 who is "behind" another consumer holding technology q_k^2 (with $q_k^1 \leqslant q_k^2$), the lagging consumer will be more likely to adopt the current technology and, as the technology changes over time, the lagging consumer will catch up with or pass the leading consumer or at least fall no further behind. In Figure 1, increasing q_k would raise the flat part of the simple NPV value function at the bottom of the stack of value functions and lift the other two value functions to preserve the order of the stack. In Figure 2, increasing q_k would lift the "cost effective" and "immediate adoption" thresholds, and the adoption regions would shrink accordingly. The bound provided in the final result of the proposition will be useful in establishing monotonicity properties in the next section.

3. Monotonicity of the Value Functions and Policies

In Figures 1 and 2, we see that in each model the value functions are increasing in the quality of the technology p_k and decreasing in the cost c_k and that the acceptance regions are monotonic in p_k and c_k; if it is optimal to adopt at one p_k (or c_k), it is also optimal to adopt at higher p_k (or lower c_k). This is always true in the simple NPV model. Moreover, the simple NPV model prescribes an unambiguous trade-off between quality and costs: changes in quality and cost will lead to increases in value and make adoption more attractive whenever these changes lead to an increase in the NPV associated with adoption.

The situation is more complicated in the single- and repeat-purchase models. Because the technology's quality and costs evolve stochastically, it is not clear when the technology will be adopted (if ever) or how long it will be held. For example, a change in the technology that improves its NPV $(((1-\delta^k)/(1-\delta))p_k - c_k)$ will make the consumer better off if she is adopting and holding the technology forever. However, if it is not optimal to adopt the technology immediately, such a change in the current technology, even if it translates directly into a change in future technologies (as it would with additive transitions of Equation (1)) may not make the consumer better off, because when she adopts, she will hold the technology for fewer than the k periods assumed in this NPV calculation. Similarly, in the repeat-purchase model, if costs are low enough, the consumer may purchase the current version of the technology now, and then purchase a new version later; again, the consumer will not enjoy the benefits of the current technology for all remaining periods. Thus, both delayed adoption and repeat adoption can lead the consumer to be "cost sensitive" and weigh adoption costs more heavily than suggested by a simple NPV calculation that assumes that the new technology would be purchased now and held forever.

3.1. Clear Improvements and Increasing Value Functions

Given that it is not clear how long the technology will be held in the single- and repeat-purchase models, to establish general monotonicity properties we need to consider a notion of a "better" technology that captures the trade-off between costs and quality and is, in some sense, flexible about how long a technology will be held. In our analysis, we will focus on a partial order on technologies that says one technology is "clearly better" than another if the net present value of the technology increases, regardless of how long the technology is held.

Definition 3.1 (Clear Improvements). We say a technology (p_k^2, c_k^2) is a *clear improvement over* (or is clearly better than) (p_k^1, c_k^1), or $(p_k^2, c_k^2) \geqslant_{\mathrm{CI}} (p_k^1, c_k^1)$ whenever one of the following conditions holds:

(i) $p_k^2 \geqslant p_k^1$ and $c_k^2 \leqslant c_k^1$. In this case, the (p_k^2, c_k^2) technology has higher quality and is cheaper than (p_k^1, c_k^1).

(ii) $p_k^2 \geqslant p_k^1$, $c_k^2 \geqslant c_k^1$ and $p_k^2 - c_k^2 \geqslant p_k^1 - c_k^1$. In this case, the (p_k^2, c_k^2) technology is more expensive than the (p_k^1, c_k^1) technology, but the increase in quality is large enough that it pays for the increase in cost in a single period.

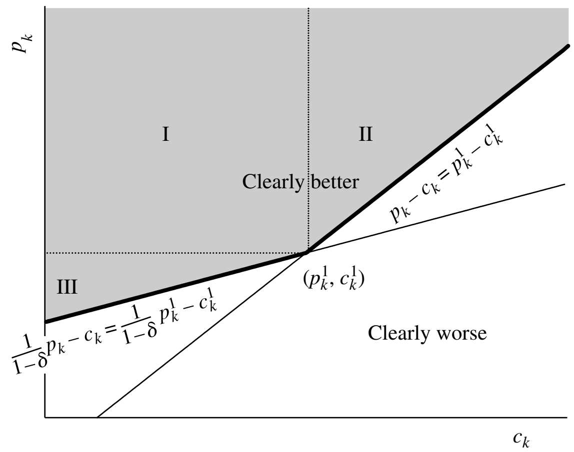
(iii) $p_k^2 \leqslant p_k^1$, $c_k^2 \leqslant c_k^1$, and $(1/(1-\delta))p_k^2 - c_k^2 \geqslant (1/(1-\delta))p_k^1 - c_k^1$. In this case, the (p_k^2, c_k^2) technology has lower quality than the (p_k^1, c_k^1) technology, but the cost reduction dominates the decrease in quality even if the technology is purchased and held forever.

Equivalently, we can say that $(p_k^2, c_k^2) \geqslant_{\mathrm{CI}} (p_k^1, c_k^1)$ if $\gamma p_k^2 - c_k^2 \geqslant \gamma p_k^1 - c_k^1$ for all γ such that $1 \leqslant \gamma \leqslant 1/(1-\delta)$. Here, γ can be interpreted as the present value multiplier for the per-period benefits p_k of the technology; $\gamma = 1$ corresponds to holding the technology for a single period and $\gamma = 1/(1-\delta)$ corresponds to holding the technology forever.[3] Figure 3 shows technologies (p_k, c_k) that are clearly better than or clearly worse than technology (p_k^1, c_k^1): the regions labeled I, II, and III in the figure correspond to the condition in the definition satisfied by these technologies. We define clearly worse technologies symmetrically, e.g., (p_k^2, c_k^2) is clearly worse than (p_k^1, c_k^1) if $(p_k^1, c_k^1) \geqslant_{\mathrm{CI}} (p_k^2, c_k^2)$.

Note that, as shown in Figure 3, there are regions of changes that are neither clearly better nor clearly worse than (p_k^1, c_k^1). For example, the wedge beneath region II in the figure represents changes in the technology that lead to an increase in the NPV of the technology if the technology were held forever (i.e., $(1/(1-\delta))p_k^2 - c_k^2 \geqslant (1/(1-\delta))p_k^1 - c_k^1$), but may not make the consumer better off if the technology is held for a shorter period. Conversely, the wedge below region III represents technologies that lead to improvements if the technology is held for a single period (i.e., $p_k^2 - c_k^2 \geqslant p_k^1 - c_k^1$), but may not lead to an improvement if the technology is held for a longer time period.

Although a clearly better technology leads to greater rewards associated with adoption, to be sure that the consumer will prefer a clearly better technology, we need to ensure that the improvement also benefits consumers who are waiting (in either the single- or repeat-purchase model) or who may adopt again in the future (in the repeat-purchase model); i.e., we need to show that the continuation values in the stochastic dynamic programming models

Figure 3. (p_k, c_k) that are clearly better or clearly worse than (p_k^1, c_k^1).



(3) and (4) increase as well as the immediate reward. To ensure that this is the case, we will assume that improvements in the current technology "persist" and translate into future improvements in future technologies. If the only uncertainty in the model were quality, we could define persistence using standard first-order stochastic dominance techniques: if we assume that the transitions are increasing in that $\tilde{p}_{k-1} \mid p_k^2$ stochastically dominates $\tilde{p}_{k-1} \mid p_k^1$ whenever $p_k^2 \geqslant p_k^1$, we would be able to show that the rewards and continuation values are both increasing in quality p_k. We can establish similar properties in this more general setting using generalized stochastic dominance techniques based on the CI partial order.

Definition 3.2 (CI-Dominance and CI-Increasing Transitions).

1. A function $u_k(p_k, c_k)$ is *CI-increasing* if $u_k(p_k^2, c_k^2) \geqslant u_k(p_k^1, c_k^1)$ whenever $(p_k^2, c_k^2) \geqslant_{\mathrm{CI}} (p_k^1, c_k^1)$.
2. $(\tilde{p}_k^2, \tilde{c}_k^2)$ *CI-dominates* $(\tilde{p}_k^1, \tilde{c}_k^1)$ (or $(\tilde{p}_k^2, \tilde{c}_k^2) \succeq_{\mathrm{CI}} (\tilde{p}_k^1, \tilde{c}_k^1)$), if $\mathbb{E}[u_k(\tilde{p}_k^2, \tilde{c}_k^2)] \geqslant \mathbb{E}[u_k(\tilde{p}_k^1, \tilde{c}_k^1)]$ for all CI-increasing functions u_k.
3. A CI-increasing set U is a set U such that if $(p_k^2, c_k^2) \geqslant_{\mathrm{CI}} (p_k^1, c_k^1)$ and $(p_k^1, c_k^1) \in U$, then $(p_k^2, c_k^2) \in U$.
4. The technology transitions are *CI-increasing*, if $(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^2, c_k^2) \succeq_{\mathrm{CI}} (\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^1, c_k^1)$ whenever $(p_k^2, c_k^2) \geqslant_{\mathrm{CI}} (p_k^1, c_k^1)$.

As with first-order stochastic dominance, there are several equivalent ways to define stochastic dominance for partial orders like the CI-order (see, e.g., Kamae et al. 1977; Müller and Stoyan 2002, pp. 81–83).

Proposition 3.3 (Equivalent Conditions for CI-Dominance). *The following conditions are equivalent*:

1. $(\tilde{p}_k^2, \tilde{c}_k^2) \succeq_{\mathrm{CI}} (\tilde{p}_k^1, \tilde{c}_k^1)$.
2. $\mathbb{E}[u_k(\tilde{p}_k^2, \tilde{c}_k^2)] \geqslant \mathbb{E}[u_k(\tilde{p}_k^1, \tilde{c}_k^1)]$ *for all CI-increasing functions* u_k.
3. $P[(\tilde{p}_k^2, \tilde{c}_k^2) \in U] \geqslant P[(\tilde{p}_k^1, \tilde{c}_k^1) \in U]$ *for any CI-increasing set* U.

4. $(\tilde{p}_k^2, \tilde{c}_k^2)$ *is equal in distribution to* $(\tilde{p}_k^1, \tilde{c}_k^1) + (\tilde{\Delta}_k^p, \tilde{\Delta}_k^c)$, *where* $(\tilde{\Delta}_k^p, \tilde{\Delta}_k^c)$ *is almost surely* (*i.e., with probability one*) *a CI-improvement on* $(0, 0)$.

The first pair of equivalent conditions simply repeats the definition of CI-dominance. The third condition is analogous to checking for first-order stochastic dominance in the univariate case by comparing cumulative probability distributions. In the final condition, we see that a CI-dominance improvement can be viewed as equivalent to adding a CI-improvement to each possible outcome.

Note that the final condition of Proposition 3.3 makes it is easy to see that the additive transitions of Equation (1) are CI-increasing. In this case, we can take the increments $(\tilde{\Delta}_k^p, \tilde{\Delta}_k^c)$ in part 4 of the proposition to be $(\tilde{\Delta}_k^p, \tilde{\Delta}_k^c) = (p_k^2, c_k^2) - (p_k^1, c_k^1)$. Thus, in this additive model, $(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^2, c_k^2)$ will CI-dominate $(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^1, c_k^1)$ whenever $(p_k^2, c_k^2) \geqslant_{\text{CI}} (p_k^1, c_k^1)$.

Similarly, consider a model like that of Balcer and Lippman (1984) (see §2.1) that distinguishes between the uncertainty associated with the occurrence of an innovation and uncertainty about the magnitude of the innovation given that it occurs: if we assume the innovations are all CI-improvements, then increasing (decreasing) the probability of an innovation occurring results in a CI-dominance improvement (worsening) in the transition function. This is also easy to see from Proposition 3.3(4).

If we assume the transitions are increasing in the sense of Definition 3.2(4), then clear improvements in the technology will "persist" and make the consumer better off in all three models.

Proposition 3.4 (CI-Increasing Value Functions). *If the technology transitions are CI-increasing, then the value functions* $v_k^*(p_k, c_k, q_k)$ *for all three models are CI-increasing for each* k *and* q_k.

Proof. We give the proof for the repeat-purchase model here; the proofs for the other models are similar and simpler. The proof is by induction. The property holds trivially for $k = 0$. Now assume $v_{k-1}^r(p_{k-1}, c_{k-1}, q_{k-1})$ is CI-increasing. Suppose (p_k^2, c_k^2) is a clear improvement over (p_k^1, c_k^1). We will consider two cases: (i) $p_k^2 \geqslant p_k^1$ and (ii) $p_k^2 \leqslant p_k^1$. First consider the case where $p_k^2 \geqslant p_k^1$. Then,

$$\begin{aligned}
&v_k^r(p_k^2, c_k^2, q_k)\\
&\quad = \max\begin{cases} p_k^2 - c_k^2 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^2) \mid p_k^2, c_k^2\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad \geqslant \max\begin{cases} p_k^2 - c_k^2 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) \mid p_k^2, c_k^2\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad \geqslant \max\begin{cases} p_k^1 - c_k^1 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) \mid p_k^1, c_k^1\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^1, c_k^1\big]\end{cases}\\
&\quad = v_k^r(p_k^1, c_k^1, q_k).
\end{aligned}$$

The first inequality follows because the value function is increasing in the quality of the technology owned (see Proposition 2.2(1)), the second inequality follows because the rewards are CI-increasing ($p_k - c_k$ if adopt, q_k if wait), and the continuation values are CI-increasing (this follows from the induction hypothesis and the assumption that the transitions are CI-increasing).

Now, suppose that $p_k^2 \leqslant p_k^1$. Then,

$$\begin{aligned}
&v_k^r(p_k^2, c_k^2, q_k)\\
&\quad = \max\begin{cases} p_k^2 - c_k^2 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^2) \mid p_k^2, c_k^2\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad = \max\begin{cases} \dfrac{1-\delta^k}{1-\delta}p_k^2 - c_k^2 + \delta\mathbb{E}\Big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^2) - \dfrac{1-\delta^{k-1}}{1-\delta}p_k^2 \,\Big|\, p_k^2, c_k^2\Big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad \geqslant \max\begin{cases} \dfrac{1-\delta^k}{1-\delta}p_k^2 - c_k^2 + \delta\mathbb{E}\Big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) - \dfrac{1-\delta^{k-1}}{1-\delta}p_k^1 \,\Big|\, p_k^2, c_k^2\Big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad \geqslant \max\begin{cases} \dfrac{1-\delta^k}{1-\delta}p_k^1 - c_k^1 + \delta\mathbb{E}\Big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) - \dfrac{1-\delta^{k-1}}{1-\delta}p_k^1 \,\Big|\, p_k^2, c_k^2\Big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad = \max\begin{cases} p_k^1 - c_k^1 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) \mid p_k^2, c_k^2\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^2, c_k^2\big]\end{cases}\\
&\quad \geqslant \max\begin{cases} p_k^1 - c_k^1 + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, p_k^1) \mid p_k^1, c_k^1\big],\\ q_k + \delta\mathbb{E}\big[v_{k-1}^r(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k) \mid p_k^1, c_k^1\big]\end{cases}\\
&\quad = v_k^r(p_k^1, c_k^1, q_k).
\end{aligned}$$

The first inequality follows because $p_k^2 \leqslant p_k^1$ and the value function satisfies Proposition 2.2(3). The second inequality follows because the NPV calculation in the reward term is CI-increasing. The last inequality follows because of the induction hypothesis and the assumption that the transitions are CI-increasing. □

Thus, with increasing transitions, we can be sure that technologies (p_k, c_k) in regions I, II, or III of Figure 3 will be preferred to technology (p_k^1, c_k^1). Similarly, we can be sure that technologies that are clearly worse than (p_k^1, c_k^1) will have lower values. Technologies that are neither clearly better nor clearly worse may or may not be preferred to (p_k^1, c_k^1), depending on the number of periods remaining (k), and/or the quality of the technology the consumer currently owns (q_k).

3.2. Monotonicity of Policies in the Single-Purchase Model

Intuitively, one might expect the optimal policies to be "increasing" in that the optimal decisions will move from waiting towards adopting if one clearly improves the technology. Although having CI-increasing transitions ensures that the value functions are increasing for clear improvements, this condition does not imply that the optimal policies will necessarily be increasing in either the single- or repeat-purchase models. The challenge is that as the current technology improves, the benefits associated with adopting and waiting both increase. Without further restrictions, the value of waiting could increase more than the value of adopting, and consequently, the consumer who would adopt with one technology may prefer to wait with a clearly better technology.

To ensure that policies are increasing in the single-purchase model, we will assume that the transitions exhibit diminishing improvements in the following sense.

Definition 3.5 (CI-Diminishing Improvements). We say the technology transitions exhibit *CI-diminishing improvements* if

$$\delta\mathbb{E}\left[\frac{1-\delta^{k-1}}{1-\delta}\tilde{p}_{k-1}-\tilde{c}_{k-1}\mid p_k, c_k\right]-\left(\frac{1-\delta^k}{1-\delta}p_k-c_k\right)$$

is CI-decreasing in (p_k, c_k).

Intuitively, this condition says that, as the current technology improves (in the CI sense), the NPV of immediate adoption $(((1-\delta^k)/(1-\delta))p_k-c_k)$ increases more than the expected NPV associated with delaying adoption one period $(\delta\mathbb{E}[((1-\delta^{k-1})/(1-\delta))\tilde{p}_{k-1}-\tilde{c}_{k-1}])$. It is not difficult to see that the additive transition model (1) satisfies this condition; see Appendix A.4 for a proof.

This diminishing improvements condition, along with increasing transitions, is sufficient to ensure that as we improve a technology, the value of adopting will improve at least as much as the value of waiting in the single-purchase model. This implies that policies are "CI-increasing" in this model.

Proposition 3.6 (Increasing Policies for the Single-Purchase Model). *Suppose technology transitions are CI-increasing and exhibit CI-diminishing improvements. Then, in the single-purchase model, if it is optimal to adopt technology (p_k^1, c_k^1), then it is also optimal to adopt any technology (p_k^2, c_k^2) such that $(p_k^2, c_k^2)\geqslant_{\mathrm{CI}}(p_k^1, c_k^1)$.*

Proof. We first show that $h_k^s(p_k, c_k, q_k)=v_k^s(p_k, c_k, q_k)-(((1-\delta^k)/(1-\delta))p_k-c_k)$ is a CI-decreasing function, for all q_k and $k\geqslant 1$; note that h_k is the difference between the value with the optimal action and the value with immediate adoption. The proof is by induction. $h_1(p_1, c_1, q_1)=\max\{q_1, p_1-c_1\}-(p_1-c_1)$ is clearly CI-decreasing. Now assume that $h_{k-1}^s(p_{k-1}, c_{k-1}, q_{k-1})$ is CI-decreasing. Then,

$$\begin{aligned}
&h_k^s(p_k, c_k, q_k)\\
&\quad=\max\Bigg\{0,\, q_k+\mathbb{E}\Bigg[\delta\left(\frac{1-\delta^{k-1}}{1-\delta}\tilde{p}_{k-1}-\tilde{c}_{k-1}\right)\\
&\qquad\qquad-\left(\frac{1-\delta^k}{1-\delta}p_k-c_k\right)\Bigg|\, p_k, c_k\Bigg]\\
&\qquad+\delta\mathbb{E}\Bigg[v_{k-1}^s(\tilde{p}_{k-1}, \tilde{c}_{k-1}, q_k)\\
&\qquad\qquad-\left(\frac{1-\delta^{k-1}}{1-\delta}\tilde{p}_{k-1}-\tilde{c}_{k-1}\right)\Bigg|\, p_k, c_k\Bigg]\Bigg\}.
\end{aligned}$$

The first expectation in the second argument inside the maximization is CI-decreasing because the transitions satisfy diminishing improvements. The second expectation is also CI-decreasing; this follows from the induction hypothesis and CI-increasing transitions. Thus, $h_k^s(p_k, c_k, q_k)$ is CI-decreasing.

Now, if it is optimal to adopt with (p_k^1, c_k^1), then $h_k^s(p_k^1, c_k^1, q_k)=0$. Because $h_k^s(p_k, c_k, q_k)$ is CI-decreasing, if $(p_k^2, c_k^2)\geqslant_{\mathrm{CI}}(p_k^1, c_k^1)$, we have $h_k^s(p_k^2, c_k^2, q_k)\leqslant 0$, which implies that it is also optimal to adopt with (p_k^2, c_k^2). $\square$

Thus, improving the current technology favors adopting in the single-purchase model. On the other hand, if we have increasing transitions and improve the future prospects for the technology without changing the current technology, this will favor waiting in the single-purchase model. That is, if we consider transitions $(\tilde{p}_{k-1}^2, \tilde{c}_{k-1}^2)\mid(p_k, c_k)$ in place of $(\tilde{p}_{k-1}^1, \tilde{c}_{k-1}^1)\mid(p_k, c_k)$ where $(\tilde{p}_{k-1}^2, \tilde{c}_{k-1}^2)\mid(p_k, c_k)\succeq_{\mathrm{CI}}(\tilde{p}_{k-1}^1, \tilde{c}_{k-1}^1)\mid(p_k, c_k)$, then the value associated with adopting is unaffected and the value associated with waiting increases. For instance, with the additive transitions of Equation (1), a CI-dominance improvement in the increments $(\tilde{u}_{k-1}^p, \tilde{u}_{k-1}^c)$ would increase the value associated with waiting and make waiting more attractive. Perhaps recognizing this issue, many firms (e.g., Apple) are reluctant to tout future improvements in their products out of fear of harming sales of their current products.

3.3. Monotonicity of Policies in the Repeat-Purchase Model

Although we might expect the policies to also be monotonic in the repeat-purchase model, the optimal policies are more complicated, and we cannot state an analogous monotonicity result. For example, the following deterministic example illustrates how improving the quality of the technology may lead the consumer to switch from adopting to waiting.

Example. Suppose there are $k=8$ periods to go, and take the quality of the consumer's technology to be $q_8=1.1$, the discount factor to be $\delta=0.99$, and the costs to be $c_k=9.5$ for all periods. Suppose the quality evolves according to $p_{k-1}=0.7p_k+3$. These transitions are CI-increasing and exhibit CI-diminishing improvements;[4] the quality level p_k will converge towards 10 over time. Here, we find that with initial quality $p_8 = 6.0$, it is optimal to adopt in the first period. However, with a higher quality level $p_8 = 6.5$, it is optimal to wait. Although this reversal may seem counterintuitive, if we look more carefully at the policies, this behavior seems reasonable. With initial quality $p_8 = 6.0$, the optimal policy calls for buying the technology in the first period and buying again in the fourth period, then holding that technology through the horizon. With initial quality $p_8 = 6.5$, the optimal policy calls for waiting in the first period, buying in the second period, and then holding that technology through the horizon.

A key feature in this example is that the magnitude of future improvements in quality is decreasing as we increase the current quality. This means that improving the current quality leads to less improvement in the future and, consequently, upgrading in the future becomes less attractive. In this example, this leads to a change in the current decision as well as the future upgrade decisions.

We can say more about the structure of the optimal policies in the special case where the transitions follow the additive model of Equation (1). Unlike the previous example, with the additive model, improvements in the current quality or cost level does not affect future improvements in cost or quality. In this case, the value function can be simplified: rather than considering the decisions to be a function of the current technology p_k, the technology-owned q_k, and cost c_k, following Balcer and Lippman (1984), we can combine p_k and q_k and consider the value as a function of the "lag" Δ_k, defined as the difference between the technology currently available and the one the consumer currently owns: $\Delta_k = p_k - q_k$. Using this, we can write the value functions as the sum of the NPV from the current technology (which depends only on q_k) and the value from future adoptions (which depends only on Δ_k and c_k):

$$v_k^r(p_k, c_k, q_k) = \frac{1-\delta^k}{1-\delta} q_k + h_k^r(p_k - q_k, c_k), \tag{5}$$

where h_k^r is defined recursively with terminal value $h_0^r(\Delta_0, c_0) = 0$ and for earlier periods,

$$h_k^r(\Delta_k, c_k) = \max \begin{cases} \dfrac{1-\delta^k}{1-\delta}\Delta_k - c_k + \delta\mathbb{E}[h_{k-1}^r(\tilde{u}_{k-1}^p, c_k + \tilde{u}_{k-1}^c)], \\ \quad \text{(if she adopts)} \\ \delta\mathbb{E}[h_{k-1}^r(\Delta_k + \tilde{u}_{k-1}^p, c_k + \tilde{u}_{k-1}^c)] \\ \quad \text{(if she does not adopt).} \end{cases} \tag{6}$$

We can show that Equation (5) holds using induction; see Appendix A.5 for a proof.

In the case of independent additive transitions, we then have the following results.

Proposition 3.7 (Repeat-Purchase Model with Additive Transisions). *Suppose the technology has additive transitions, as in Equation* (1). *Then, in the repeat-purchase model*:

1. *The value function* $h_k^r(\Delta_k, c_k)$ *of Equation* (6) *is CI-increasing in* (Δ_k, c_k).
2. *For any fixed* c_k, *the optimal policy is increasing in* Δ_k: *if it is optimal to adopt with lag* Δ_k^1, *it is also optimal to adopt with any* $\Delta_k^2 \geqslant \Delta_k^1$.
3. *If the quality of the technology* p_k *is nondecreasing over time* (*i.e.*, $\tilde{p}_{k-1} \geqslant p_k$ *almost surely for all* k), *then the optimal policies are CI-increasing*: *if it is optimal to adopt with* (Δ_k^1, c_k^1), *it is also optimal to adopt with any* (Δ_k^2, c_k^2) *such that* $(\Delta_k^2, c_k^2) \geqslant_{\mathrm{CI}} (\Delta_k^1, c_k^1)$.

The first result follows from Proposition 3.4 after recalling that the additive transitions are CI-increasing. The second result shows that the policies are monotonic in the lag Δ_k with a fixed cost c_k or, equivalently, monotonic in quality p_k, with fixed technology owned q_k and cost c_k. This generalizes the threshold policy result of Balcer and Lippman (1984) to the case with uncertainty about costs and the possibility of decreases in quality, but says nothing about simultaneous changes in quality and costs.

The last part of Proposition 3.7 considers simultaneous changes in quality and costs, but imposes the additional requirement that quality be nondecreasing. The following example shows that if quality can decrease, the optimal policies for the repeat-purchase model may not be monotonic in costs.

Example. Let us reconsider the example of §2.4 with $k = 3$ periods remaining, discount factor $\delta = 1$, where the consumer owns a technology with quality $q_3 = 1$ and the new technology evolves deterministically with qualities $p_k = 2.8, 2.0$, and 3.0 for $k = 3, 2, 1$, respectively. We assume that the costs decrease by 3 after the first period ($c_2 = c_3 - 3.0$) and then remain constant ($c_1 = c_2$). These transitions are additive, as in Equation (1), but with time-varying increments. With $c_3 = 3.0$, we have the example of §2.4 and find that it is optimal to not adopt in the first period in the repeat-purchase model. However, with higher initial costs $c_3 = 4.0$, the optimal policy calls for buying in the first period. Although this reversal may seem counterintuitive, if we look at the whole sequence of decisions, this result makes sense: with initial costs $c_3 = 3.0$, the optimal policy calls for waiting in the first period and buying in the next two periods. With the higher initial costs of $c_3 = 4.0$, the optimal policy calls for buying in the first period and not buying in the next two periods. Thus, as one might expect, increasing costs leads to fewer total purchases even though the change from waiting to buying in the first period decision is perhaps counterintuitive when viewed in isolation.

The last of part of Proposition 3.7 relies on the following lemma.

Lemma 3.8. *Suppose that the transitions satisfy the additive model* (1) *and the quality of the technology is nondecreasing over time* (*i.e.,* $\tilde{p}_{k-1} \geqslant p_k$ *almost surely for all* k). *Let* $q_k^2 \geqslant q_k^1$. *Then*

(1) $v_k^r(p_k, c_k, q_k^2) - v_k^r(p_k, c_k, q_k^1)$ *is increasing in* c_k; *and*

(2) $v_k^r(p_k, c_k, q_k^2) - v_k^r(p_k, c_k, q_k^1) - c_k$ *is decreasing in* c_k.

The proof of this lemma is rather involved, but the intuition is as follows. The difference $v_k^r(p_k, c_k, q_k^2) - v_k^r(p_k, c_k, q_k^1)$ is the difference in values for consumers holding technology q_k^2 and q_k^1, respectively; the consumer holding q_k^1 is lagging the other in that $q_k^1 \leqslant q_k^2$. As discussed following Proposition 2.2, as the technology changes over time, the lagging consumer will be next to adopt or else both will adopt at the same time. Now, if the technology does not get worse over time, once the lagging consumer adopts, she will either "leapfrog" the other consumer and become the new leader or else the two consumers will be tied. When the former laggard becomes the leader, the former leader will be the next to adopt, thereby continuing this game of leapfrog (or "keeping up with the Joneses"). As this process continues, the original lagging consumer will have adopted either the same number of times as the original leader or once more than the original leader. For this reason, the difference $v_k^r(p_k, c_k, q_k^2) - v_k^r(p_k, c_k, q_k^1)$ is increasing in cost (increasing the cost hurts the lagging consumer most) and $v_k^r(p_k, c_k, q_k^2) - v_k^r(p_k, c_k, q_k^1) - c_k$ is decreasing in cost (the negative impact on the laggard due to increasing the adoption costs is less than the cost of adopting one extra time).

These comparisons between leaders and laggards arise when studying policies and comparing continuation values associated with buying and waiting. In the example above that shows optimal policies may not be monotonic in costs, if the consumer buys in the first period, she holds that technology for all remaining periods. If she waits in the first period, she will adopt multiple times in the future. Thus, in this setting, increasing the cost of adoption has a bigger impact on the consumer who waits. However, having nondecreasing quality ensures that the waiting and adopting consumers will play leapfrog with alternating adoptions, and increasing costs will not make adopting more attractive than waiting.[5]

4. Convexity and Increases in Uncertainty

In this section, we investigate the convexity properties of the value functions and optimal adoption and rejection regions and the impact of increasing uncertainty on policies and values. In the simple NPV model, the value function is jointly convex in quality and costs, (p_k, c_k). The adoption and rejection regions are separated by a line $(((1-\delta^k)/(1-\delta))p_k - c_k = ((1-\delta^k)/(1-\delta))q_k)$, and therefore both regions are convex in (p_k, c_k).

The situation is more complex in the single- and repeat-purchase models. As in our study of monotonicity properties, our analysis of convexity properties relies on stochastic dominance arguments.

Definition 4.1 (Convex Dominance and Convex Transitions).

1. $(\tilde{p}_k^2, \tilde{c}_k^2)$ *convex-dominates* $(\tilde{p}_k^1, \tilde{c}_k^1)$ (or $(\tilde{p}_k^2, \tilde{c}_k^2) \succeq_{CX} (\tilde{p}_k^1, \tilde{c}_k^1)$), if $\mathbb{E}[u_k(\tilde{p}_k^2, \tilde{c}_k^2)] \geqslant \mathbb{E}[u_k(\tilde{p}_k^1, \tilde{c}_k^1)]$ for all convex functions u_k.
2. The technology transitions are *convex* if

$$\mathbb{E}\big[u_{k-1}(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k, c_k)\big]$$

is a convex function of (p_k, c_k) for all convex functions $u_{k-1}(p_{k-1}, c_{k-1})$.

This definition of convex dominance is fairly standard and is applied here in a multivariate context; see, e.g., Müller and Stoyan (2002) or Shaked and Shanthikumar (2007). In the univariate context, a random variable (or distribution) that convex dominates another random variable (or distribution) is viewed as "more uncertain" or "more spread out." A similar interpretation holds in this context as well: $(\tilde{p}_k^2, \tilde{c}_k^2) \succeq_{CX} (\tilde{p}_k^1, \tilde{c}_k^1)$ is equivalent to saying that $(\tilde{p}_k^2, \tilde{c}_k^2)$ is equal in distribution to $(\tilde{p}_k^1, \tilde{c}_k^1) + (\tilde{\Delta}_k^p, \tilde{\Delta}_k^c)$ where $\mathbb{E}[(\tilde{\Delta}_k^p, \tilde{\Delta}_k^c) \mid (\tilde{p}_k^1, \tilde{c}_k^1)] = (0, 0)$ (see, e.g., Müller and Stoyan 2002). In this representation, $(\tilde{\Delta}_k^p, \tilde{\Delta}_k^c)$ is analogous to a mean-preserving spread in the univariate setting. The convexity condition on transitions is analogous to the CI-increasing condition on transitions in Definition 3.2 and will be used to ensure that the convexity properties persist through the recursive structure of the single- and repeat-purchase models.

For example, with the additive model of (1), if the increments are independent, an increase in the uncertainty in the increments $\tilde{u}_{k-1}^p$ and $\tilde{u}_{k-1}^c$ in the usual univariate sense would lead to a convex dominance increase in $(\tilde{p}_{k-1}, \tilde{c}_{k-1})$. It is not difficult to see that the additive transitions of Equation (1) are convex. With additive transitions, for any convex function $u_{k-1}(p_{k-1}, c_{k-1})$ we have

$$\begin{aligned}
&\alpha\mathbb{E}\big[u_{k-1}(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^1, c_k^1)\big] \\
&\qquad + (1-\alpha)\mathbb{E}\big[u_{k-1}(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^2, c_k^2)\big] \\
&\quad = \mathbb{E}\big[\alpha u_{k-1}(p_k^1 + \tilde{u}_{k-1}^p, c_k^1 + \tilde{u}_{k-1}^c) \\
&\qquad\qquad + (1-\alpha) u_{k-1}(p_k^2 + \tilde{u}_{k-1}^p, c_k^2 + \tilde{u}_{k-1}^c)\big] \\
&\quad \leqslant \mathbb{E}\big[u_{k-1}(p_k^\alpha + \tilde{u}_{k-1}^p, c_k^\alpha + \tilde{u}_{k-1}^c)\big] \\
&\quad = \mathbb{E}\big[u_{k-1}(\tilde{p}_{k-1}, \tilde{c}_{k-1}) \mid (p_k^\alpha, c_k^\alpha)\big],
\end{aligned}$$

where $(p_k^\alpha, c_k^\alpha) = \alpha(p_k^1, c_k^1) + (1-\alpha)(p_k^2, c_k^2)$ for some α such that $0 \leqslant \alpha \leqslant 1$. The inequality above follows from the fact that $u_{k-1}(p_{k-1}, c_{k-1})$ is assumed to be convex.

If the technology transitions are convex, then the value function for the single-purchase model is convex in (p_k, c_k) and the adoption regions are convex.

Proposition 4.2. *If the technology transitions are convex, then in the single-purchase model, for each k and* q_k:

1. *The value function* $v_k^s(p_k, c_k, q_k)$ *is convex in* (p_k, c_k); *and*
2. *The adoption regions are convex: If it is optimal to adopt technologies* (p_k^1, c_k^1) *and* (p_k^2, c_k^2), *then it is also optimal to adopt technology* $(p_k^\alpha, c_k^\alpha) = \alpha(p_k^1, c_k^1) + (1-\alpha)\cdot(p_k^2, c_k^2)$ *for any* α *such that* $0 \leqslant \alpha \leqslant 1$.

In the real options literature, this convexity of the value function is often interpreted as meaning "uncertainty is good." Here a similar interpretation holds in that a consumer would rather take a gamble on the technology than take a sure technology with the expected quality and costs for sure: if the gamble has probability α of yielding technology (p_k^1, c_k^1) and probability $1-\alpha$ of technology (p_k^2, c_k^2), the gamble yields an expected value, $\alpha v_k^s(p_k^1, c_k^1, q_k) + (1-\alpha)v_k^s(p_k^2, c_k^2, q_k)$, that is higher than the value $v_k^s(p_k^\alpha, c_k^\alpha, q_k)$ with the corresponding "expected technology" $(p_k^\alpha, c_k^\alpha) = \alpha(p_k^1, c_k^1) + (1-\alpha)(p_k^2, c_k^2)$. The intuition behind this is the standard argument in the real options literature: if the gamble turns out well, you can exercise the option to adopt the technology; if it doesn't turn out well, you can decide not to adopt.

Moreover, if we have convex transitions and increase uncertainty about future prospects without changing the current technology, this will make the consumer better off and will favor waiting in the single-purchase model. In other words, if we consider transitions $(\tilde{p}_{k-1}^2, \tilde{c}_{k-1}^2) \mid (p_k, c_k)$ instead of $(\tilde{p}_{k-1}^1, \tilde{c}_{k-1}^1) \mid (p_k, c_k)$ where $(\tilde{p}_{k-1}^2, \tilde{c}_{k-1}^2) \mid (p_k, c_k) \succeq_{CX} (\tilde{p}_{k-1}^1, \tilde{c}_{k-1}^1) \mid (p_k, c_k)$, then the value associated with adoption is not affected and the value associated with waiting increases. Thus, although consumers should appreciate increases in uncertainty about future technologies, producers should not deliberately induce uncertainty about future technologies' quality and costs because this uncertainty encourages the consumer to wait to adopt. Thus, in the single-purchase model, CI-improvements in the forecasts for future technologies (as discussed in §3.2) and increases in uncertainty in these forecasts both make consumers better off, but encourage waiting.

We cannot state a similar convexity result for the repeat-purchase model for general technology transitions. However, if the transitions follow the additive model of Equation (1), we can show that the single- and repeat-purchase value functions are both jointly convex in (p_k, c_k, q_k) (earlier we showed that the single-purchase value function is convex in (p_k, c_k) for a given q_k).

Proposition 4.3. *If technology transitions follow the additive model of Equation* (1), *then* $v_k^s(p_k, c_k, q_k)$ *and* $v_k^r(p_k, c_k, q_k)$ *are both jointly convex in* (p_k, c_k, q_k).

Thus, in the case with additive transitions, we can say that "uncertainty is good" in the repeat-purchase model as well as the single-purchase model.

Although the repeat-purchase value function is convex with additive transitions, this does not imply that the corresponding adoption regions will be convex. Even in the additive model of Equation (1) with positive quality increments, it is not difficult to find examples where both the adoption and rejection regions are nonconvex. Our earlier monotonicity results for the additive case (Proposition 3.7(c)) imply that whenever (p_k^1, c_k^1) and (p_k^2, c_k^2) are CI-ordered and it is optimal to adopt (or reject) at both (p_k^1, c_k^1) and (p_k^2, c_k^2), it will also be optimal to adopt (or reject) for all convex combinations of these two technologies. However, when (p_k^1, c_k^1) and (p_k^2, c_k^2) are not CI-ordered, the optimal policies may be nonmonotonic as we combine (p_k^1, c_k^1) and (p_k^2, c_k^2) (i.e., as we vary α in $\alpha(p_k^1, c_k^1) + (1-\alpha)(p_k^2, c_k^2)$), and nonconvexities may arise.

The careful reader may note that in Figure 2b, the adoption regions for the single-purchase model do not appear to be convex, even though the transitions for the example approximate the additive model (1), which is stochastically convex. The transitions for the example are, however, truncated to fit on a grid and, in particular, the costs are constrained to be nonnegative. As a result of this, the transitions are not stochastically convex, and the adoption region for the single-purchase model is not convex, as they would be if the transitions were truly additive. Similarly, the value function for the repeat-purchase model shown in Figure 1 is visibly nonconvex for low costs, for the same reason.

5. Conclusion

One main conclusion from our analysis is that, given an appropriate notion of a "better" technology—for example, the notion of a clear improvement—most of the natural properties of the single-purchase model hold in the case where both costs and quality are uncertain, with fairly mild assumptions on transitions: the value functions and policies both have natural monotonic and convexity properties in this setting.

The analogous results are, however, much more delicate in the repeat-purchase setting. Although the value functions satisfy natural monotonicity assumptions, we can establish monotonicity results for policies only in the case where transitions are additive and when there is no possibility of decreasing quality. These monotonicity results generalize results for models with deterministic costs (e.g., from Balcer and Lippman 1984) to the case with stochastic costs. However, the assumptions required to ensure monotonic policies highlight the fragility of these results in the repeat-purchase setting. As illustrated in our examples, in the repeat-purchase setting it is quite possible for a decrease in quality or an increase in cost to lead the consumer to purchase a technology she would have otherwise declined. Although this behavior may seem counterintuitive at first, when we inspect the optimal policies more carefully, this behavior seems entirely reasonable.

As discussed in §2.2, we view the repeat-purchase model to be the most realistic and appropriate of the three models. Unfortunately, it is also the most difficult to analyze. The differences in policies across these models (e.g., compare Figures 2a and 2b with Figure 2c) suggest that the simple NPV and single-purchase models may not be suitable proxies for the repeat-purchase model. Although the repeat-purchase model is the most difficult of the three models to solve, there do not appear to be any easy "short cuts" for solving these problems. We need to think carefully about the timing of adoption and the possibility of upgrades when considering technology adoption problems; these decisions may interact in complicated ways.

There are two natural directions to extend the analysis of the paper. The first natural extension would be to consider multiple technologies. Given multiple technologies, we could take the single technology in our analysis to be the "best" of the available technologies. However the notion of the "best" technology could depend on the dynamics of all of the available technologies. For example, as discussed in §2.4, the presence of a low-cost, rapidly improving technology may make the consumer reluctant to invest in a higher-cost, high-quality technology. Christensen (1997) calls such cheap, low-quality but improving technologies "disruptive technologies" (or "disruptive innovations") and provides a number of examples of disruptive technologies that have radically changed the marketplace. It would be interesting to formally study the dynamics of the cost-quality trade-off in a setting with competing products that have different rates and kinds of technological improvements.

A second natural extension would be to introduce uncertainty about the value of the technology itself. As discussed in the introduction, there is a stream of literature that focuses on the role of uncertainty about the quality or profitability of the current technology, rather than assuming (as we do here) that the quality of the technology is fully observed. It would be interesting to extend this uncertainty analysis to consider uncertainty in costs as well as quality while incorporating the possibility of repeat purchases.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Endnotes

1. Balcer and Lippman (1984) allow the probability of an innovation occurring to depend on the time since the last innovation as well as a "discovery potential" variable that changes stochastically whenever an innovation occurs.
2. If adoption has negative costs ($c_k < 0$), then a technology that pays for itself immediately (i.e., satisfies $p_k - c_k \geqslant q_k$) may be worse than the technology owned (i.e., $p_k < q_k$), and adoption may be rejected in the repeat-purchase model.
3. It is possible to have a weaker order by letting γ be such that $1 \leqslant \gamma \leqslant ((1-\delta^k)/(1-\delta))$. In this time-dependent order, instead of holding the technology forever, the consumer is required to hold the technology for all the remaining k periods. The results for this time-dependent order are similar, but the analysis is more cumbersome.
4. We discuss this form of transition in more detail in Appendix A.8. To be clear about the transitions for costs, we can take $c_{k-1} = 0.7c_k + 2.85$. If we start with a cost of 9.5, the costs will remain at 9.5.
5. Although we have focused on improvements defined in terms of the CI-order, we can establish similar monotonicity results using other partial orders instead. These alternative orders may allow us to study transitions that are not CI-increasing or do not exhibit CI-diminishing returns. See Appendix A.8 for more details.

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References

Balcer, Y., S. A. Lippman. 1984. Technological expectations and adoption of improved technology. *J. Econom. Theory* **34**(2) 292–318.

Bertsekas, D. P. 1995. *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont, MA.

Cho, S., K. McCardle. 2009. The adoption of multiple dependent technologies. *Oper. Res.* **58**(1) 157–169.

Christensen, C. 1997. *The Innovator's Dilemma*: *When New Technologies Cause Great Firms to Fail*. Harvard Business School Press, Cambridge, MA.

Dixit, A. K., R. S. Pindyck. 1994. *Investment Under Uncertainty*. Princeton University Press, Princeton, NJ.

Doraszelski, U. 2001. The net present value method versus the option value of waiting: A note on Farzin et al. (1998). *J. Econom. Dynam. Control* **25**(8) 1109–1115.

Doraszelski, U. 2004. Innovations, improvements, and the optimal adoption of technologies. *J. Econom. Dynam. Control* **28**(7) 1461–1480.

Farzin, Y. H., K. J. M. Huisman, P. M. Kort. 1998. Optimal timing of technology adoption. *J. Econom. Dynam. Control* **22**(5) 779–799.

Jensen, R. 1982. Adoption and diffusion of an innovation of uncertain profitability. *J. Econom. Theory* **27**(1) 182–193.

Kamae, T., U. Krengel, G. L. O'Brien. 1977. Stochastic inequalities on partially ordered spaces. *Ann. Probab.* **5**(6) 899–912.

Kornish, L. 1999. On optimal replacement thresholds with technological expectations. *J. Econom. Theory* **89**(2) 261–266.

Lippman, S. A. 1975. On dynamic programming with unbounded rewards. *Management Sci.* **21**(11) 1225–1233.

Lippman, S. A., K. F. McCardle. 1987. Does cheaper, faster, or better imply sooner in the timing of innovation decisions? *Management Sci.* **33**(8) 1058–1064.

McCardle, K. F. 1985. Information acquisition and the adoption of new technology. *Management Sci.* **31**(11) 1372–1389.

Müller, A., D. Stoyan. 2002. *Comparison Methods for Stochastic Models and Risks*. Wiley, Chichester, UK.

Schumpeter, J. A. 1934. *The Theory of Economic Development*. Harvard University Press, Cambridge, MA.

Shaked, M., J. G. Shanthikumar. 2007. *Stochastic Orders*. Springer, New York.

Smith, J. E., K. F. McCardle. 2002. Structural properties of stochastic dynamic programs. *Oper. Res.* **50**(5) 796–809.

Stokey, N. L., R. E. Lucas. 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA.

Ulu, C., J. E. Smith. 2009. Uncertainty, information acquisition, and technology adoption. *Oper. Res.* **57**(3) 740–752.