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Online Supplement to Uncertainty, Information Acquisition, and Technology Adoption

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A Additional Proofs

A.1 Proof of Proposition 3.4

Proof. (i) Notice that $f(\pi_2) \succeq_{LR} f(\pi_1)$ is equivalent to $f(x_1;\pi_1)f(x_2;\pi_2) - f(x_1;\pi_2)f(x_2;\pi_1) \geq 0$ for $x_2 \geq x_1$. Then

$$\begin{aligned}
&f(x_1;\pi_1)f(x_2;\pi_2) - f(x_1;\pi_2)f(x_2;\pi_1) \\
&\qquad= \int_{\theta_1}\int_{\theta_2} \left(L(x_1|\theta_1)L(x_2|\theta_2) - L(x_1|\theta_2)L(x_2|\theta_1)\right)\pi_2(\theta_2)\pi_1(\theta_1)\, d\theta_2\, d\theta_1 \\
&\qquad= \int_{\theta_1}\int_{\theta_2\geq\theta_1} \left(L(x_1|\theta_1)L(x_2|\theta_2) - L(x_1|\theta_2)L(x_2|\theta_1)\right)\pi_2(\theta_2)\pi_1(\theta_1)\, d\theta_2\, d\theta_1 \\
&\qquad\quad+ \int_{\theta_1}\int_{\theta_2\leq\theta_1} \left(L(x_1|\theta_1)L(x_2|\theta_2) - L(x_1|\theta_2)L(x_2|\theta_1)\right)\pi_2(\theta_2)\pi_1(\theta_1)\, d\theta_2\, d\theta_1
\end{aligned}$$

Reversing the order of integration and interchanging the symbols θ_1 and θ_2, this last integral may be rewritten as:

$$\int_{\theta_1}\int_{\theta_2\geq\theta_1} \left(L(x_1|\theta_2)L(x_2|\theta_1) - L(x_1|\theta_1)L(x_2|\theta_2)\right)\pi_2(\theta_1)\pi_1(\theta_2)\, d\theta_2\, d\theta_1$$

Combining this with the first integral above, we have

$$\begin{aligned}
&f(x_1;\pi_1)f(x_2;\pi_2) - f(x_1;\pi_2)f(x_2;\pi_1) \\
&\quad= \textstyle\int_{\theta_1}\int_{\theta_2\geq\theta_1} \left(L(x_1|\theta_1)L(x_2|\theta_2) - L(x_1|\theta_2)L(x_2|\theta_1)\right)\left(\pi_2(\theta_2)\pi_1(\theta_1) - \pi_2(\theta_1)\pi_1(\theta_2)\right)\, d\theta_2\, d\theta_1.
\end{aligned}$$

The assumption that $L(x|\theta)$ satisfies the MLR property implies that $(L(x_1|\theta_1)L(x_2|\theta_2) - L(x_1|\theta_2)L(x_2|\theta_1))$ is nonnegative and the assumption that $\pi_2 \succeq_{LR} \pi_1$ implies that $(\pi_2(\theta_2)\pi_1(\theta_1) - \pi_2(\theta_1)\pi_1(\theta_2))$ is nonnegative. Thus $f(x_1;\pi_1)f(x_2;\pi_2) - f(x_1;\pi_2)f(x_2;\pi_1)$ is nonnegative.

(ii) By Definition 3.1, proving $\Pi(\pi, x_2) \succeq_{LR} \Pi(\pi, x_1)$ is equivalent to proving that for $\theta_2 \geq \theta_1$,

$$\frac{\Pi(\theta_2;\pi,x_2)}{\Pi(\theta_2;\pi,x_1)} \geq \frac{\Pi(\theta_1;\pi,x_2)}{\Pi(\theta_1;\pi,x_1)}$$

Writing this out using Bayes' rule and canceling common terms, we see that this is equivalent to $L(x|\theta_2) \succeq_{LR} L(x|\theta_1)$. □

A.2 Proof of Proposition 4.1

Proof. Let $W^n = E[\theta|\mathbf{x}_n]$ denote the expected value of θ after starting with prior π and observing a sequence $\mathbf{x}_n$ of n signals. W^n is a martingale (see Billingsley (1986) p. 483). By an application of Doob's Martingale convergence theorem (see, e.g., Theorem 35.5 in Billingsley (1986) p. 492), W^n almost certainly converges to a random variable $W^\infty = E[\theta|\mathbf{x}_\infty]$ which is the expected benefit after seeing an infinite sequence of signals.

Let $Y^n = \max\{0, E[\theta - K|\mathbf{x}_n]\} = \max\{0, W^n - K\}$ denote the expected value if forced to adopt or reject after seeing n observations. Since Y^n is a continuous function of W^n, Y^n almost certainly converges to $Y^\infty = \max\{0, W^\infty - K\}$.

Finally let $Z^n = E[\max\{0, W^\infty - K\}|\mathbf{x}_n]$. This is the expected value with an infinite sequence of signals; it is analogous to the expected value with perfect information and represents the expected value if the consumer could somehow observe an infinite sequence of signals and then decide whether to adopt or reject the technology. Like W^n, Z^n is a martingale and converges to $Z^\infty = E[\max\{0, W^\infty - K\}|\mathbf{x}_\infty] = \max\{0, W^\infty - K\} = Y^\infty$.

(i) Y^n is the value if the consumer adopts or rejects after seeing n observations. Since Z^n is the expected value of infinite number of costless observations, the value of gathering additional information is less than $-c + \delta Z^n$. Since Y^n and Z^n almost certainly converge to a common value, for these convergent signal sequences, when $c > 0$, for large enough n we will have $\delta Z^n - Y^n < c$ and at this point (if not sooner) it will be optimal to adopt or reject without gathering additional information.

(ii) If $c = 0$, it will never be (uniquely) optimal to reject. In this setting, the value of gathering additional information is less than δZ^n. Adopting will certainly be optimal if $Y^n > \delta Z^n$. If $\delta < 1$ and adopting is truly optimal ($W^\infty > K$), then $Y^\infty = Z^\infty = \max\{0, W^\infty - K\} = Y^\infty$ is positive and, for almost all signal sequences, for large enough n we will have $Y^n > \delta Z^n$.

(iii) If $c \geq 0$, the finite-horizon value function $v_k^*(\pi)$ is increasing in k and bounded above by the expected value of perfect information, $E[\max\{0, \theta - K\}]$. This bound is finite because of our assumption that θ is π-integrable. Such a bounded monotonic sequence must converge. □

A.3 Proof of Proposition 5.1

Proof. Because the adoption and rejection regions will vary depending on prior signals, it is helpful to make this dependence explicit in the recursive construction of these functions. Let $\mathbf{x}_k$ denote the vector of signals observed before period k and let $b_k(\theta, \mathbf{x}_k)$, $p_k(\theta, \mathbf{x}_k)$ and $c_k(\theta, \mathbf{x}_k)$ denote the functions involved in the proposition.

With one period to go, $c_1(\theta, \mathbf{x}_1) = 0$ for all $\mathbf{x}_1$ and $p_1(\theta, \mathbf{x}_1) = 1$ or 0 depending on whether the policy P calls for adoption or rejection after observing $\mathbf{x}_1$. In the recursive construction, assume that for period $k-1$ we can write

$$v_{k-1}(\pi; \mathbf{x}_{k-1}, P) = \int_\theta b_{k-1}(\theta; \mathbf{x}_{k-1})\pi(\theta)\, d\theta$$

where $b_{k-1}(\theta; \mathbf{x}_{k-1}) = (\theta - K)p_{k-1}(\theta; \mathbf{x}_{k-1}) - c_{k-1}(\theta; \mathbf{x}_{k-1})$.

In period k, if the policy P calls for adopting or rejecting, then the value function with k periods to go is either 0, $\int_\theta \theta\pi(\theta)d\theta$ and the choices for b_k, p_k and c_k are obvious. If the policy P calls for continuing, then

$$v_k(\pi;\mathbf{x}_k,P) = -c+\delta\left(\int_{x_k\in A_k(\mathbf{x}_k)}\left(\int_\theta(\theta-K)\Pi(\theta;\pi,x_k)\,d\theta\right)f(x_k;\pi)\,dx_k\right.$$
$$\left.+\int_{x_k\in C_k(\mathbf{x}_k)}\left(-c+\delta\int_\theta b_{k-1}(\theta;\mathbf{x}_{k-1})\Pi(\theta;\pi,x_k)\,d\theta\right)f(x_k;\pi)\,dx_k\right),$$

where $A_k(\mathbf{x}_k)$ and $C_k(\mathbf{x}_k)$ denote policy P's adoption and continuation regions in period $k-1$ in the scenario in which $\mathbf{x}_k$ has been previously observed. Rewriting $b_{k-1}(\theta;\mathbf{x}_{k-1})$ as $(\theta-K)p_{k-1}(\theta;\mathbf{x}_{k-1})-c_{k-1}(\theta;\mathbf{x}_{k-1})$ and canceling the $f(\pi)$ term appearing in the denominator of the posterior distribution and as an integrand, we can rewrite this as:

$$v_k(\pi;\mathbf{x}_k,P) = -c+\delta\left(\int_\theta(\theta-K)\left(\int_{x_k\in A_k(\mathbf{x}_k)}L(x_k|\theta)\,dx_k\right)\pi(\theta)\,d\theta\right.$$
$$\left.+\int_\theta\left(\int_{x_k\in C_k(\mathbf{x}_k)}(-c+\delta\left((\theta-K)p_{k-1}(\theta;\mathbf{x}_{k-1})-c_{k-1}(\theta;\mathbf{x}_{k-1})\right)L(x_k|\theta)\,dx_k\right)\pi(\theta)\,d\theta\right)$$

Collecting terms, we can write the value function in the case where the policy P calls for waiting in the current period as

$$v_k(\pi;\mathbf{x}_k,P) = \int_\theta b_k(\theta;\mathbf{x}_k)\pi(\theta)\,d\theta$$

where $b_k(\theta;\mathbf{x}_k)=(\theta-K)p_k(\theta;\mathbf{x}_k)-c_k(\theta;\mathbf{x}_k)$ and

$$p_k(\theta;\mathbf{x}_k) = \int_{x_k} Q_k(x_k,\theta;\mathbf{x}_k)L(x_k|\theta)\,dx_k$$

$$c_k(\theta;\mathbf{x}_k) = \int_{x_k} R_k(x_k,\theta;\mathbf{x}_k)L(x_k|\theta)\,dx_k$$

where

$$Q_k(x_k,\theta;\mathbf{x}_k) = \begin{cases}\delta, & x_k\in A_k(\mathbf{x}_k)\\ \delta p_{k-1}(\theta;\mathbf{x}_{k-1}), & x_k\in C_k(\mathbf{x}_k)\\ 0, & \text{otherwise.}\end{cases}$$

$$R_k(x_k,\theta;\mathbf{x}_k) = \begin{cases}c, & x_k\in A_k(\mathbf{x}_k)\\ c+\delta c_{k-1}(\theta;\mathbf{x}_{k-1}), & x_k\in C_k(\mathbf{x}_k)\\ c, & \text{otherwise.}\end{cases}$$

From the recursive definitions, it is easy to see that $0\le p_k(\theta;\mathbf{x}_k)\le 1$ and that $c_k(\theta;\mathbf{x}_k)\ge 0$ if $c\ge 0$. We can also see that p_k and c_k can be interpreted as the discounted probability of adoption and expected discounted costs.

We now show that the probability of adoption is increasing in its arguments if the signal process satisfies the MLR property and P is a monotonic policy. If the policy P calls for immediate adoption or rejection, then $p_k(\theta;\mathbf{x}_k)=1$ or 0, respectively which are trivially increasing in θ. We will thus focus on the case where the policy P calls for waiting in the current period and show that $p_k(\theta;\mathbf{x}_k)$ is increasing componentwise in θ and $\mathbf{x}_k$ using an inductive proof. First note that $p_1(\theta;\mathbf{x}_1)$ is equal to 0 or 1 depending on whether the last signal x_1 is above or below the final adoption threshold $x_1^a(\mathbf{x}_1)$; by our assumption that the policy P is monotonic, this threshold is decreasing in the earlier signals.

Now proceeding by induction, assume that $p_{k-1}(\theta; \mathbf{x}_{k-1})$ is increasing componentwise in its arguments. Note that $Q_k(x_k, \theta; \mathbf{x}_k)$ is increasing componentwise in its arguments because of this induction hypothesis and the fact that the adoption and rejection thresholds are decreasing in $\mathbf{x}_k$ and $0 \leq p_{k-1}(\theta; \mathbf{x}_{k-1}) \leq 1$. Then, for $\theta_2 \geq \theta_1$ and $\mathbf{x}_k \geq \mathbf{y}_k$ (where "$\geq$" is the componentwise increasing order on the signals), we have

$$\begin{aligned} p_k(\theta_2; \mathbf{x}_k) = \int_{x_k} Q_k(x_k, \theta_2; \mathbf{x}_k) L(x_k|\theta_2)\, dx_k &\geq \int_{x_k} Q_k(x_k, \theta_2; \mathbf{y}_k) L(x_k|\theta_2)\, dx_k \\ \geq \int_{x_k} Q_k(x_k, \theta_1; \mathbf{y}_k) L(x_k|\theta_2)\, dx_k &\geq \int_{x_k} Q_k(x_k, \theta_1; \mathbf{y}_k) L(x_k|\theta_1)\, dx_k = p_k(\theta_1; \mathbf{y}_k) \end{aligned}$$

where the first inequality follows because $Q_k(x_k, \theta; \mathbf{x}_k)$ is increasing in $\mathbf{x}_k$. The second inequality follows because $Q_k(x_k, \theta; \mathbf{x}_k)$ is increasing in θ for each $(x_k, \mathbf{x}_k)$. The next inequality follows because $Q_k(x_k, \theta; \mathbf{x}_k)$ is an increasing function of x_k for each $(\theta; \mathbf{x}_k)$ and $L(x_k|\theta)$ is a totally positive kernel of order 2 (TP2) and the "monotonicity preserving property" of TP2 kernels (see Karlin 1968, Proposition 3.1, p. 22). Thus the adoption probability $p_k(\theta; \mathbf{x}_k)$ is increasing in θ as well as the previously observed signals $\mathbf{x}_k$.

If the technology is evolving over time according to conditional probability distribution $g(\theta_{k-1}|\theta_k)$ (see §8), the proof goes through assuming $g(\theta_{k-1}|\theta_k)$ has the MLR property and using the "basic composition theorem" in Karlin (1968). □

A.4 Proof of Lemma 6.1

Proof. The posterior distribution $\Pi(\pi_\alpha, x)$ for a given x can be written as,

$$\begin{aligned} \Pi(\theta; \pi_\alpha, x) &= \frac{L(x|\theta)\pi_\alpha(\theta)}{f(x; \pi_\alpha)} = \alpha \frac{L(x|\theta)\pi_1(\theta)}{f(x; \pi_\alpha)} + (1-\alpha) \frac{L(x|\theta)\pi_2(\theta)}{f(x; \pi_\alpha)} \\ &= \beta(x)\Pi(\theta; \pi_1, x) + (1 - \beta(x))\Pi(\theta; \pi_2, x) \end{aligned}$$

where $\beta(x) = \alpha \frac{f(x;\pi_1)}{f(x;\pi_\alpha)}$ and $1-\beta(x) = (1-\alpha)\frac{f(x;\pi_2)}{f(x;\pi_\alpha)}$. For convex functions u defined on distributions on θ, we have

$$\begin{aligned} E\left[u(\Pi(\pi_\alpha, \tilde{x}_\alpha))\right] &= \int_x u(\Pi(\pi_\alpha, x)) f(x; \pi_\alpha) dx \\ &= \int_x u\left(\beta(x)\Pi(\pi_1, x) + (1-\beta(x))\Pi(\pi_2, x)\right) f(x; \pi_\alpha) dx \\ &\leq \int_x \left(\beta(x) u\left(\Pi(\pi_1, x)\right) + (1-\beta(x)) u\left(\Pi(\pi_2, x)\right)\right) f(x; \pi_\alpha) dx \\ &= \alpha \int_x u\left(\Pi(\pi_1, x)\right) f(x; \pi_1) dx + (1-\alpha) \int_x u\left(\Pi(\pi_2, x)\right) f(x; \pi_2) dx \\ &= \alpha E\left[u(\Pi(\pi_1, \tilde{x}_1))\right] + (1-\alpha) E\left[u(\Pi(\pi_2, \tilde{x}_2))\right] \end{aligned}$$

The inequality follows because u is convex and the next equality follows by canceling the denominator of the $\beta(x)$ and $(1 - \beta(x))$ terms. □

A.5 Proof of Proposition 7.1

Proof. (i) Let $v_k^*(\pi; c)$ denote the value function with cost c; we show that this function increases with decreases in c. Following the proof strategy used in the proof of Proposition 3.6 and Lemma 3.7, we note that the rewards associated with adopting or rejecting do not depend on c. The Bayesian updating process is unaffected by changes in c; so if $v_{k-1}^*(\pi; c)$ increases with decreases in c, so does the value associated with continuing. Thus the value functions $v_k^*(\pi; c)$, as the maximum of three functions that are (at least weakly) increasing with decreases c, is also increasing with decreases in c.

(ii) Because the value from adoption and rejection actions are not affected by c and the value from gathering information increases, the information gathering region must expand.

(iii) The decision to adopt or reject is made when the consumer's distribution π first exits the information gathering region. These distribution paths and their probabilities are unaffected by changes in c. Thus, expanding the information gathering regions must delay the exit from the region and delays the adoption or rejection decision. □

A.6 Proof of Proposition 7.3

Let $v_k^*(\pi; X)$ denote the value function with signal process X. Our first goal is to show that the value function is increasing in the informativeness of the signal process, that is, $X \succeq_B Y$ implies $v_k^*(\pi; X) \geq v_k^*(\pi; Y)$. Blackwell (1951) proved this result (and the converse) for single period decision problems; see Crémer(1982) for a simple proof of Blackwell's result. Marschak and Miyasawa (1968) provide numerous related results, also focusing on single period decision problems. Here we apply these ideas in a dynamic programming setting using a recursive argument.

Following the proof strategy used in the proof of Proposition 3.6 and Proposition 6.2, we first establish a lemma that shows that the continuation value is "increasing in the informativeness of the signal process" if the continuation value function is convex; the convexity of the value functions v_k^* was established in Proposition 6.2. Marschak and Miyasawa (1968)'s results imply this lemma (combining their Theorems 12.1 and 9.8) for discrete systems; we provide a simple direct proof here for completeness.

Lemma A.1. *If $X \succeq_B Y$ then $E\left[u(\Pi(\pi, \tilde{x}))\right] \geq E\left[u(\Pi(\pi, \tilde{y}))\right]$ for functions $u(\pi)$ that are convex in π.*

Proof. By definition, $X \succeq_B Y$ implies that there exists a random variable Y^* that had the same likelihood function as Y and is a garbling of X. By construction, then $E\left[u(\Pi(\pi, \tilde{y}^*))\right] = E\left[u(\Pi(\pi, \tilde{y}))\right]$. Let $f(x|y^*)$ be the conditional probability distribution for x conditional on values of y^*. Now consider a fixed y^*,

$$\Pi(\theta; \pi, y^*) \quad = \quad \int_x \Pi(\theta; \pi, \{x, y^*\}) f(x|y^*)\, dx = \int_x \Pi(\theta; \pi, x) f(x|y^*)\, dx.$$

The first equality follows from the "law of total probability" and the second from the fact that the posterior given both signals x and y^*, $\Pi(\pi, \{x, y^*\})$ is equal to $\Pi(\pi, x)$ when y^* is a garbling of x. Using this result with Jensen's inequality, for convex functions u,

$$u(\Pi(\pi, y^*)) \leq \int_x u(\Pi(\pi, x)) f(x|y^*)\, dx.$$

Since this holds for all y^*, we have

$$E\left[u(\Pi(\pi,\tilde{y}^*))\right] = \int_{y^*} u(\Pi(\pi,y^*))f(y^*)\,dy^* \le \int_{y^*}\int_x u(\Pi(\pi,x))f(x|y^*)f(y^*)\,dx\,dy^*$$
$$= \int_x u(\Pi(\pi,x))\left(\int_{y^*} f(x|y^*)f(y^*)\,dy^*\right)\,dx = \int_x u(\Pi(\pi,x))f(x)\,dx = E\left[u(\Pi(\pi,\tilde{x}))\right]$$

Thus $E\left[u(\Pi(\pi,\tilde{x}))\right] \ge E\left[u(\Pi(\pi,\tilde{y}))\right]$ for convex u. □

Proof. **Proposition 7.3** (i) With this lemma in hand, we can now use a standard recursive argument to show that $X \succeq_B Y$ implies $v_k^*(\pi;X) \ge v_k^*(\pi;Y)$. First, note the terminal value functions are $v_0^*(\pi;X) = v_0^*(\pi;Y) = 0$ and thus (weakly) satisfy this condition. Now suppose that $v_{k-1}^*(\pi;X) \ge v_{k-1}^*(\pi;Y)$. To show that $v_k^*(\pi;X) \ge v_k^*(\pi;Y)$, we note that the rewards associated with adopting or rejecting do not depend on the signal process. Similarly the cost c of information is assumed to be the same for X and Y and the lemma implies that the value of continuing is greater with X than Y. Then for each action, the value is (weakly) larger with signal process X than signal process Y and thus $v_k^*(\pi;X) \ge v_k^*(\pi;Y)$.

(ii) To see that the information gathering region expands, note that the value from rejecting and adopting are not affected by the informativeness of the sampling process and the value of gathering information increases. □

A.7 Proof of Proposition 7.4

Proof. (i) Let $v_k^*(\pi;K)$ denote the value function with cost K. The proof that $v_k^*(\pi;K)$ increases with decreases in K is similar to the proof of Proposition 7.1.

(ii) We now show that decreasing the adoption cost expands the adoption region. Define $g_k(\pi,K) = v_k^*(\pi;K) + K$. It is straightforward to show that $g_k(\pi,K)$ is increasing in K. Let $K_2 \ge K_1$, and suppose it is optimal to adopt with π and K_2, then, $g_k(\pi,K_2) = \int_\theta \theta\pi(\theta)d\theta$. To show that it is optimal to adopt given K_1, we need to show that $g_k(\pi,K_1) \le \int_\theta \theta\pi(\theta)d\theta$; this follows from the fact that $g_k(\pi,K)$ is increasing in K.

(iii) If it were optimal to reject with π and K_1, then $v_k^*(\pi,K_1) = 0$. Since $v_k^*(\pi,K)$ is decreasing in K, then if $K_2 \ge K_1$, $v_k^*(\pi,K_2) \le 0$ which implies it is also optimal to reject given K_2. Thus increasing K expands the rejection region; decreasing K has the opposite effect.

(iv) The decision to adopt is made when the consumer's distribution π first enters the adoption region. These distributions evolve over time with the sequence distributions and the probabilities of these sequences are unaffected by changes in K. Because the adoption region has expanded, any path that has entered the adoption region with a higher adoption cost has already entered the adoption region with a lower adoption cost. Taking expectations over all such paths proves that the probability of adoption increases and the expected time until adoption decreases. □

A.8 Proof of Proposition 8.1

Proof. The proof of (i) is analogous to the proof of Lemma 3.7, taking into account the technology transitions. We can write $g_k(\pi) = v_k^*(\pi) - \int_{\theta_k} \theta_k\pi(\theta_k)d\theta_k$ as a dynamic programming recursion

$g_0(\pi) = -\int_{\theta_0} \theta_0 \pi(\theta_0) d\theta_0$, and for $k > 0$,

$$g_k(\pi) = \max\Big\{ -\int_{\theta_k} \theta_k \pi(\theta_k) d\theta_k, -K,$$
$$-c - (1-\delta)\int_{\theta_k} \theta_k \pi(\theta_k) d\theta_k + \delta \int_{\theta_k} E[\theta_{k-1} - \theta_k | \theta_k] \pi(\theta_k) d\theta_k + \delta E[g_{k-1}(\Pi(\pi, \tilde{x}_{k-1}))] \Big\}$$

In the dynamic programming definition of $g_k(\pi)$, because of the assumption on the decreasing rate of improvement, the reward functions are LR decreasing for each action. The generalization of Lemma 3.5 implies that the transitions preserve this property. (ii) Given (i), the proof of the monotonicity of policies follows exactly as before. □

A.9 Proof of Proposition 8.2

Proof. The dynamic programming rewards are unaffected by the changes in the technology transitions. We will show that waiting improves with improved technology transitions by showing that $\eta_2(\pi) \succeq_{LR} \eta_1(\pi)$ where η_i is the next-period prior with technology process g_i. The assumption of an LR-improving technology process is equivalent to requiring $g_i(\theta_k|\theta_k - 1)$ to be totally positive of order 2 (TP2) in each pair of arguments i, θ_k and θ_{k-1} with the third held constant. By Theorem 5.1 Karlin (1968; p. 123), this implies that $\eta_i(\theta_{k-1}; \pi)$ is TP2 in i and θ_{k-1}, which is equivalent to $\eta_2(\pi) \succeq_{LR} \eta_1(\pi)$. Given that the signal process satisfies the MLR property, this then implies LR dominance among the corresponding signal distributions and the posterior distributions for each signal. With the additional assumption that $g_i(\theta_k|\theta_{k-1})$ satisfies the MLR property, we know that the next period value function, v^*_{k-1}, is LR increasing. Taken together, this implies the expected continuation value, $E[v^*_{k-1}(\Pi(\pi, \tilde{x}))]$, is greater with the LR-dominant technology transitions. Thus, waiting becomes more attractive, the value function weakly improves, and the information gathering region expands. □

References

[1] Billingsley, P. 1986. *Probability and Measure.* Wiley, New York.

[2] Crémer, J. 1982. A simple proof of Blackwell's "Comparison of experiments" theorem. *Journal of Economic Theory* **27**(2) 439–443.

[3] Karlin, S. 1968. *Total Positivity, Vol.I.* Stanford University Press, Stanford, CA.

[4] Marschak, J., Miyasawa, K. 1968. Economic comparability of information systems. *International Economic Review* **9**(2) 137–174.